Link-arm mooring stability study

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Summary
In this presentation, an alternative method for ship mooring, using link arms between the ship side and wharf, is introduced. It will noted, that under certain conditions, this mechanism may fail due to lost stability, resulting into possible accident or loss of usability. This may take place if sideways forces, such as wind load exist, pushing the ship towards the wharf.

This phenomenon is addressed and simulated by using ABAQUS Standard FEA-package with nonlinear hydrostatic pressure loading, discrete rigid bodies and connector elements. Ship stability curves are also calculated with the same method and verified against the results obtained with commonly used naval architectural NAPA program.
1 Introduction

Some types of modern ships are docked for a prolonged period of time, sometimes for several years or even decades. Examples of such special ships are floating hotels, storage barges etc. Standard means of docking with ropes and sleepers/buffers is not practical in such cases, typically requiring constant power input for rope tightening equipment, among other issues, such as a possibility of vandalism.

The mooring method should be able to allow changes in water levels due to, for example, tidal changes or storm-driven flooding. Typically the method should also allow some change in ship roll and trim angles due to wave action and changes in ship load configuration.

In the past, there have been some quite successful methods to provide long term mooring, such as connecting the ship to the poles anchored to seabed with loops attached to ship side. However, if the ship is ever to be repositioned to some other location, this method is not practical.

In this document an alternative method for mooring that has been proposed, using rigid link arms is introduced and evaluated against stability.

2 Nomenclature

The layout and nomenclature of the proposed mooring method is shown in figure 1.

- ASL Ship’s cross section at sea level plane
- b Ship breadth
- g Gravitational constant (9.81 m/s²)
- CG Ship centre of gravity
- CP Wind centre of pressure
- L Length, distance
- m Ship mass
- M (MC) Metacentre about ship’s axis
- X,Y,Z  Fixed coordinates
- x,y,z  Ship coordinates
- \( \alpha \) Link arm angle
- \( \beta \) Trim angle about y-axis
- \( \phi \) Roll (listing) angle about x-axis
- \( \theta \) Heading angle about z-axis
- \( \rho \) Water density (sea water = 1025 kg/m³)
- \( \rho_{\text{air}} \) Air density (1.225 kg/m³)

The ship is connected to the wharf with three rigid arms being hinged at both ends, two connected to wharf pivot A from ship pivots C and D and one rigid arm between pivots B and D. With this configuration, ship translations in XY-plane and about Z-axis are constrained. For a freely floating ship, the buoyancy loads balance the ship vertically and about Y and Z axis rotations, provided that the ship is statically stable (metacentre M is above ship’s centre of gravity CG). Configuration allows water level changes and variation in trim and roll angles about y and x axis correspondingly.

Wind load is assumed to affect through wind center of pressure CP.

3 Stability in side wind

3.1 Freely floating ship

For a free ship in the open sea, the wind load affecting through the CP will cause rolling moment as the sideways support is provided below sea level by water friction and pressure redistribution as the ship moves sideways. This rolling moment is resisted by the ship’s righting moment until the maximum righting moment is exceeded, where after the ship capsizes. Typically, the wind speed when this would happen is very high.

3.2 Ship connected with rigid links to wharf

In this case, the balancing force against the wind is provided by the link arms. When the arms are leaned due to the water level or turning due to the difference in height between wind centre of pressure and the link arms, the arm forces have vertical component that is balanced with ship’s buoyant reactions. At some wind speed, the arm angles will become too large and they cannot provide the lateral support any more. The ship will run into the wharf or to the sea bottom.

It should be noted, that in this case at the stability limit, the ship roll angle does not need to be as large as is the case with freely floating ship, whose maximum righting moment should, according to the international rules, take place with roll angles in excess of 25°. Typical roll angle for maximum moment is much higher, however.
4 Rigid arm stability analysis

Generally, the ship buoyancy force and righting moments are highly nonlinear functions of ship draft, trim and roll angles (even though they often can be linearized about analysis point). Naturally, these are also special for each ship shape. Doing analysis in the nonlinear domain with traditional methods would be very tedious.

However, some modern FEA packages, such as Abaqus 6.8-1 Standard provide the necessary tools to carry out the complete stability analysis. One of the most beneficial features in this respect is the possibility to apply nonlinear hydrostatic pressure on the surfaces in geometrically nonlinear analyses. Hydrostatic pressure affecting the outer surfaces has the form

\[
p_{hs} = \begin{cases} 
\rho g (z_0 - z), & z < z_0 \\
0, & z \geq z_0
\end{cases}
\]

Here \( \rho \) is the fluid density, \( g \) gravitational acceleration, \( z \) the height of the point in question and \( z_0 \) the height at fluid surface level.

We are interested only of the ship’s rigid body translations and rotations, and therefore we model the ship surfaces with discrete rigid mesh. Rigid mesh results into a rigid body whose displacements and rotations are completely defined by six degrees of freedom located at the rigid body’s reference point.

The ship flexibility should be taken into consideration in real life, but in this demonstration we satisfy with rigid body simplification. When ship’s structural details are known, it is straightforward to add actual structural response behaviour with standard flexible element mesh as required.

Ship rigid body’s reference point is positioned at its calculated centre of gravity. Separately modelled rudder and link arms’ ship connection nodes are connected to ship CG with rigid MPC beam-type connector elements. These connectors eliminate the dependent degrees of freedom while forcing them to follow the master nodes. Rigid links between ship connection nodes and fixed mooring nodes at wharf are modelled with link-type connector elements with Lagrangian formulation. Wind centre of pressure node was connected to the CG with Lagrange type rigid beams. Lagrangian connector elements in Abaqus realize rigid body constraints exactly by using Lagrange multipliers and are therefore suitable also in cases, when load or displacement constraint is applied on the slave node. All connector elements are applicable in both small and large displacement/rotation cases.

4.1 Solution stabilization

In the static equilibrium, the buoyant forces provide the necessary balancing forces required to oppose the force imbalance that is left after link arm support reactions have been applied. However, the buoyant reactions are realised by using nonlinear hydrostatic pressure, affecting only the load vector in solution. Since the corresponding terms in the system’s stiffness matrix are zero, the system is singular and not soluble by direct static analysis.

There are a few methods to circumvent this problem; one is to apply dynamic analysis, where inertial acceleration forces stabilize the solution. Smooth, slow loading should then be used along with long integration time to yield an approximately (pseudo)static response. Damping could also be used, but the viscous dissipation would cause an additional error to the results.

In static solution, the stiffness matrix can be made stable by introducing artificial stabilizing springs to the system. This may be done by adding a vertical Z-spring and rotational springs about X-and Y-axis between earth and ship’s CG.
The spring coefficients should be large enough to avoid singularity problems, but on the other hand, small enough to yield accurate enough results. Initially, spring coefficients were given values equal to about 1/1000 of the buoyant force gradients at freely floating configuration.

4.1.1 Ship buoyancy gradients

These gradients were used to calculate suitable stabilizing spring coefficients in static analysis.

Ship’s buoyancy force gradient in vertical direction is equal to the change of water mass x gravity per change of depth, or

$$\frac{dF_z}{dz} = A_{SL} \rho g \tag{2}$$

Here $A_{SL}$ is the ship’s cross-sectional area in water level with the current load condition; $\rho$ is the sea water density 1025 kg/m$^3$ and $g$ gravitational acceleration 9.81 m/s$^2$. It is easy to see that by integrating (2) along $z$, we get the familiar Archimedes law, i.e., the lifting force is equal to the weight of the displaced water volume.

Ship’s moment gradient about the longitudinal axis is equal to the ship’s mass force times the vertical distance between ship’s centre of gravity and ship metacentre $M_x$ about x-axis. Metacentre is the point through which the moments caused by the hydrostatic forces vanish. In order to be stable, the moment gradient must be positive, i.e. ship’s metacentre must be above the centre of gravity. The position of the metacentre at the analysis load condition can be read from, for example, NAPA output. NAPA is a state-of-the-art, general ship initial design and analysis tool that is used extensively in the ship industry.

The equation for righting moment gradient is therefore

$$\frac{dM_x}{d\phi} = mg\left(M_x - CG\right)_z \tag{3}$$

For a long and slender ship, the position of metacentre and moment gradient about y-axis are mostly determined by the second moment of the ship’s cross section at sea level about the y-axis. Assuming approximately rectangular cross section at sea level, we get

$$\frac{dM_y}{d\beta} \approx \frac{1}{12} \rho gbL_{WL}^3 \tag{4}$$

Here $L_{WL}$ is the water line length and $b$ breadth at the sea level.

These spring values can be used as the first linear approximation of the buoyant stiffness, while nonlinear effects are ignored. However, in the current case, we use them merely as the magnitude guideline for analysis stabilization spring coefficients.

4.2 Analyses

4.2.1 Static stability model

This analysis was done for verification. In this case, we calculate the righting moment against increasing roll angle with static analysis and compare the result with the stability curve obtained from NAPA. In this case, the link arms are removed. In the first static step, we start by forcing the ship to move to some floating position with forced displacements with active hydrostatic pressure and gravity. In the second step, the uz- and ry-displacements are released and ship model sets into the normal floating position. In the third step the roll angle about x-axis is gradually increased with forced rotation at CG node and the corresponding reaction moment is evaluated.
4.2.2 Link arm model

In the second analysis, the ship stability is evaluated when it is attached to the wharf with rigid links. The ship model is first positioned into the static equilibrium position with static steps similarly as with the static stability model with gravity and hydrostatic loads. During these first two static steps, the arms are kept in position with soft support springs. The third step is replaced with implicit dynamic stability step. All ship constraints are released and the link arm land pivots are now fixed into their current position. We apply increasing forced displacement in the y-direction on the wind CP –node. Displacement is used instead of point load since this approach is more stable and works even when the opposing load starts to decrease. Using Riks or arc-length method instead is also possible. In order to avoid dynamic effects, we use smoothed load amplitude during the long, 100 second step time. In order to use dynamic solution, ship must have inertial mass moments modelled.

5 Numerical example

In this demonstration we are using the geometry of “NapaStar”, NAPA Corporation’s fictional freighter that is used extensively for training purposes with NAPA-program. NAPA is a state-of-the-art, general ship initial design and analysis tool that is used extensively in the ship industry. Even though NapaStar does not represent any probable ship to be used with rigid link arm mooring, it has the advantage of having a realistic ship shape and thoroughly analysed stability properties. The current load case is a realistic case for a load that is rigidly tied to the ship hull with empty fuel tanks so that no cargo or free fluid surface shifts can take place.

The ship and rudder skin was obtained as IGES surface written by NAPA. Deck details and thruster tunnel were then added manually in Abaqus CAE.

![Figure 3 Side plan of Napastar at CL](image)

We assume the following loading condition:
- \( x_{CG}=41.16 \text{m from aft perpendicular (APP)} \),
- \( z_{CG}=4.95 \text{m above baseline (ABL)} \),
- \( y_{CG}=0 \text{m (at centreline)} \),
- \( x_{CP}=33.9 \text{m APP (NAPA)} \),
- \( z_{CP}=8.738 \text{m ABL} \),
- Wind area above SL at analysis load condition \( A_{w}=370.8 \text{ m}^2 \) (NAPA)
- Vertical distance between CP and water CP at design state 5.9m
- Total displacement 5107 metric tonnes.
- Breadth \( b=13 \text{m} \)
- Total length 87m
- Water line length \( L_{WL} \) approximately 70m

We use general mesh density of 1m. Cargo hatches are assumed to be watertight, but deckhouse structure is not (deckhouse structures are not included in the model otherwise but in wind surface area calculation).
Figure 4 NapaStar analysis model mesh seen from starboard

Rudder is a separate rigid body, which is connected with the hull model with rigid beam type constraints. The model has 5100 discrete rigid shell elements and 5200 nodes. Hydrostatic pressure is applied on all outer surfaces.

5.1 Static stability curve

Static stability curve was obtained with the static stability model for validating the analysis method against NAPA stability results. Reaction moments were calculated for increasing roll angles. Finally, the righting arm was calculated by dividing the moment with ship’s weight. In figure 5, the now calculated result (blue line) is compared with the result obtained with NAPA (red line). At angles below 40 degrees, the curves match almost perfectly. Over 40 degree angles, the currently calculated curve falls below NAPA curve, with maximum error of about 9mm in righting arm.

Figure 5 Comparison of stability curves calculated by NAPA and Abaqus
5.1.1 Convergence with mesh size

The result accuracy depends on the discrete mesh size. In order to evaluate the effect of an increased mesh density, a test run with otherwise similar model with 0.5m mean element side length was done and reaction moment difference against 1m mesh results was calculated for different roll angles. The result is shown in figure 7.

The difference is insignificant (<3mm) and 1m mesh is accurate enough for engineering purposes in the current case. Slight deviation visible at high roll angles between current analysis and NAPA results cannot be explained by too coarse mesh. It is more probable that the difference is caused by slightly simplified front deck shape in the current analysis that makes difference when the deck submerges.
However, the analysis model is valid for stability purposes with engineering accuracy and at low roll angles the curve match is very good.

Strain energy (due to artificial springs) was less than 1% of the total work done by gravity and rolling torque.

5.2 Link arm stability analysis

Link arms are assumed to be positioned at line along deck corner (y=6.5m CL, z=7.09m ABL). Foremost link x=69.8m APP, and aft link x’s are 15m and 20m APP. Link arms are supposed to be in horizontal position with the analysis loads without wind. In this case, the front arm is perpendicular to both ship and wharf edge and aft arms form right angle between each other and 45 degrees to x-axis. Distance from wharf edge is 2.5m. Soft springs (k~1kN/m) are used to keep the link arms in the right position during the first step with hydrostatic pressure applied. After this, the analysis continues from the second step as described in chapter 4.2.2.

Rotational inertias are required for dynamic step procedure. These values were approximated by assuming the ship to be of prismatic shape and constant density. Dynamic effects were minimised by using smooth amplitude for forced displacement during a long (100 second) load step, that is over 10 times longer than the ship’s longest fundamental period about y-axis.

![Figure 8 Total, strain and kinetic energy during the stability step](image)

In figure 8, the system’s total energy is compared with the strain energy introduced by the stabilizing springs and the kinetic energy due to the ship movements in half logarithmic scale. It can be seen that strain energy is less than 1% of the total energy and kinetic energy less than 0.1% of the total energy so the bias is acceptable in engineering terms.
5.2.1 Results for link arm analysis

The peak wind force is 817 kN with 13 degree roll angle (see figure 9). Assuming wind area of 370.8 m², air density of 1.225 and ship’s drag coefficient of 1.3 (assuming flat plate profile with aspect ratio of 10³ and ignoring ship’s roll angle effects) we get from wind load equation

\[ F_W = \frac{1}{2} C_D \rho V^2 A_W \]  

the critical wind speed of 52.6 m/s or 189 km/hr for the current configuration.

In comparison, the capsizing wind force for a freely floating ship would be, with assumed 5.9m moment arm between air CP and water CP, 12.68 MNm/5.9m = 2150 kN, or 2.6 times higher than the critical load with link arm mooring. Above-mentioned moment arm is given for zero roll angle, but actual moment arm should be calculated for the ship at its peak righting moment roll angle.

![Stability curve with link arms](image)

*Figure 9 Wind force vs. wind CP horizontal displacement*

The critical wind velocity is in this case very high and is not of great concern when the ship is located in a sheltered harbour. However, in this example the ship wind area and CP location are low when compared to a typical floating hotel with multiple decks. Depending of the actual link arm configuration and orientation, ship load case and ship’s cross-wind properties, the link arm stability may be an issue and should be checked in advance.

6 Acknowledgements

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7 Literature