Experiences with Embedded Elements in Tire Modeling

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Abstract: The addition of embedded elements to the stable of capabilities within ABAQUS can potentially improve solution times due to a reduction in the number of elements needed in the model, while still giving reasonably accurate results. The tire, as a layered composite structure, can use the embedded element technique to advantage in modeling. Multiple layers of ply components through the thickness of the tire, represented by rebars attached to membrane elements, can be embedded into a single host solid element, representing the rubber. In the conventional, non-embedded model, one solid element is necessary between each layer of ply components, increasing the number of elements required in the model. A 30% reduction in the total number of elements is possible with the use of embedded elements while maintaining the accuracy of the desired results.

This paper describes our experiences with embedded elements in various types of tire modeling. The technique has been used in both ABAQUS/Standard and ABAQUS/Explicit analyses. It shows nearly equivalent results at the macroscopic level at reduced computational expense compared to values from the conventional, non-embedded model for analyses performed in Standard. A difference in convergence behavior is noted between the two models, and depends on the loading conditions being applied. Examples from the different loading conditions are shown. The results from an Explicit analysis are again nearly identical compared to the non-embedded model, but the improvement in computational speed is not realized. This is due to the decrease in stable time increment caused by the added constraints in the embedded element technique. A discussion on potential future implementation needs is offered in the light of the tire analyses performed.

Keywords: Tires, Embedded Element, ABAQUS/Standard, ABAQUS/Explicit

1. Introduction

Finite element analysis has been used in tire design for many years. The increased capabilities in commercial finite element programs, such as ABAQUS, and increased speed in computer
hardware has led to the modeling of many performance characteristics. Some of this work has moved from the hands of the finite element specialist to the tire development engineer, who uses the analysis product as a turnkey system. To be most effective in impacting the tire design, the analysis must be robust and quick running so that the engineer can trust the results and receive them in a timely fashion.

Making this goal difficult is the complex makeup of the tire and the inherent nonlinearity in the material properties and loading. Except for the tread pattern, the tire appears to be a relatively simple structure – round and black. However, inside it is made up of layers of cords embedded in many different formulations of rubber. Figure 1 shows a cut away of a typical passenger car tire. The cords in the belt and body layers are oriented at selected angles to give the desired response to loading. Each cord is made up of many filaments twisted together. This twist produces a nonlinear response of the cords as the load is applied, and this characterization is important in some analyses to improve the dimensional prediction or convergence behavior. Many different rubber compounds are used in the components of the tire to give desired performance. The moduli of these compounds vary from soft to very stiff. The rubber is inherently nonlinear and is typically represented by material models such as Mooney-Rivlin or Ogden. In some analyses, an approximate linear representation is adequate.

The internal geometric complexity shown in Figure 1 must be modeled in order to predict many of the important performance features required from a tire analysis. Typically, a combination of solid elements (CGAX4x, C3D8x, CCL12x) and membrane elements (MGAX1, M3D4x, MCL6x) are used to represent the different components of the tire. Rebars are attached to the membrane elements to provide the stiffness and directional response of the cord layers. A solid element lies between the membrane cord layers to give the required spacing. A typical mesh of half the tire cross-section is shown in Figure 2. The detailed tread pattern is neglected in this model, being replaced by solid circumferential ribs. A model of this type can predict global behavior as well.

![Figure 1. Cutaway view of a typical passenger car tire](image-url)
as internal strain states not associated with the tread pattern. Including the tread pattern into the model increases the complexity and size, and is required for the adequate prediction of certain performance. Because of the additional detail present in the model, run times can be long. This is especially true in transient dynamic analyses.

This amount of detail in the model is not always needed to predict the desired tire performance. Sometimes, only a global response is needed and so a simpler representation of the tire could provide a more economical solution. This is the case if only the stiffness of the tire is desired. Another example is the prediction of the interaction between the tire and the road surface to give an assessment of wear potential. To do this, the detailed strain information within the tire cross-section is not needed. The non-tread region of the model only needs to adequately represent the stiffness response of that portion of the tire. Additional detail can then be put into the tread region of the model to better capture the interface response.

The embedded element capability in ABAQUS provides a means to simplify the tire model while still capturing the structural response. Multiple cord layers (represented by membrane elements with rebars) can be embedded into a single layer of solid elements. An example embedded element mesh of a passenger car tire half cross-section is shown in Figure 3. This is the same tire shown in Figure 2. Two solid elements are used through the thickness of the non-tread portion of the cross-section as host elements. The modulus of the host element is determined by averaging the moduli of the different rubber components present in the area covered by the element. The different shadings of the host elements in Figure 3 represent different averaged rubber properties. The tread region mesh and the membrane elements representing the cord layers are the same in both the standard and embedded mesh models. Comparing the meshes in Figures 2 and 3 shows the savings in elements using the embedded element technique. The reduction in the total number of elements in the model leads to a more economical solution while maintaining the accuracy of the global response.
One drawback to modeling the tire with embedded elements is the loss of some detail in the internal strain state. This is especially true at the edges of the cord layers, where shear strains dominate the strain response. Embedding the cord layers constrains the shearing action so that the strain levels are not fully developed. If this response is important, the detailed mesh must be retained in that portion of the model. Embedded elements could still be used away from these cord ending locations to produce a more economical solution.

The embedded element technique has been used in a number of different tire modeling applications, using both ABAQUS/Standard and ABAQUS/Explicit. The next sections will give examples from applications using both programs. Comparisons between the performance of the embedded mesh model and the standard mesh model will be made. The benefits and limitations of the embedded element technique for tire analysis will also be pointed out.

2. ABAQUS/Standard Applications

ABAQUS/Standard is used primarily to model the static and steady state rolling responses of the tire. The desired output from these analyses ranges from a simple global stiffness response to detailed strain and energy values within the tire. The shape and pressure distribution of the contact patch between the tire and road surface is also an important predicted response.

The performance of models made with the standard and embedded meshing techniques will be shown for a typical passenger car tire and a medium radial truck tire. The internal construction and load conditions are different for the two tires, so additional insight can be gained from the comparison between the standard mesh and embedded mesh of the two tires.

2.1 Passenger car tire

The cross-section meshes for the passenger car tire used in the evaluation have already been shown in Figures 2 and 3. As noted above, the membrane/rebar mesh representing the cord layers
is the same in the two models. The solid element representation makes the difference in the models. A typical tire analysis is run in successive phases starting with an axisymmetric model to impose the rim constraints and apply the inflation pressure. Half the tire cross-section with symmetric boundary conditions is typically used for efficiency. The model is then revolved to make a 360° three-dimensional model of half the cross-section. Standard elements (C3D8x and M3D4x) are used in the refined mesh region where contact with the road will occur. Cylindrical elements (CCL12x and MCL6x) are used outside this potential road contact region for model efficiency (Kennedy, 2003). The same circumferential mesh refinement is used in both the standard mesh and embedded mesh models. The number of elements and degrees of freedom for each revolved half model are listed in Table 1. The embedded mesh model has approximately 25% less elements, but 10% more degrees of freedom.

### Table 1. Half model data for passenger car tire in ABAQUS/Standard.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Standard Mesh</th>
<th>Embedded Mesh</th>
<th>Embedded/Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of elements</td>
<td>12820</td>
<td>9803</td>
<td>0.765</td>
</tr>
<tr>
<td>Number of degrees of freedom</td>
<td>40440</td>
<td>44697</td>
<td>1.105</td>
</tr>
<tr>
<td>Cpu time for vertical loading phase (sec)</td>
<td>931</td>
<td>748</td>
<td>0.803</td>
</tr>
<tr>
<td>Number of iterations (sdi, equil)</td>
<td>29, 29</td>
<td>33, 30</td>
<td>1.086</td>
</tr>
<tr>
<td>Cpu time (sec) per iteration</td>
<td>16.05</td>
<td>11.87</td>
<td>0.740</td>
</tr>
<tr>
<td>Vertical stiffness (N/mm)</td>
<td>242.1</td>
<td>241.6</td>
<td>0.998</td>
</tr>
</tbody>
</table>

A vertical load is applied to the model to simulate the deformation of the tire against the road surface. In the final phase of the analysis, the model is reflected about a line to produce a full three-dimensional representation of the tire. A steady-state rolling velocity can then be applied with cornering or camber, if desired. Results of both standard mesh and embedded mesh models will be shown for each of these loading steps.

Figure 4 shows the inflated shapes of the tire models overlaid on top of each other. The solid line gives the results from the embedded mesh model and the dashed line is from the standard mesh model. There is little difference in the predicted inflated shape from the two models.

The contact between the tire and the road due to vertical loading is shown in Figure 5 for the two models. The results are shown from the full 3D model rather than from the half cross-section 3D model for ease of visualization. The contact shape and pressure distributions are nearly the same for the two models. Table 1 lists the run times and vertical stiffness values for the two meshed models. The stiffness of the two models is comparable. The embedded mesh model takes about 20% less time to run than the standard mesh model. Part of this difference in run time is due to the more efficient solution of the embedded mesh model, and part is due to differences in the convergence behavior of the models. The embedded element model required four more severe discontinuity iterations (sdi) and one additional equilibrium iteration (equil). To determine the efficiency of the embedded element technique on a per iteration basis, the solution time is divided by the number of iterations and listed in the table. The embedded element model solves 26% faster than the standard mesh model using this criterion.
A free rolling condition is applied to the full 3D model of the tire using the Steady State Transport capability. This is followed by the application of 1° cornering. Table 2 lists the data for this portion of the model run. The model sizes are in the same ratio as the half model values listed in Table 1, as expected. The run times, however, are much improved for the embedded mesh model compared to the standard mesh model – now over 45% faster. Again this is improvement is due to the difference in convergence behavior of the two models and the solution efficiency of the embedded mesh model. As listed in the table, there are six additional equilibrium iterations required for the standard mesh model solution and the same number of severe discontinuity iterations. On a per iteration basis, the embedded mesh model is about 40% more efficient.

The resulting contact shape and pressure distribution for the two models are nearly identical for the two rolling conditions. The contact plots for the straight-ahead rolling condition are shown in Figure 6 for the standard mesh and embedded mesh models. Results for the two models undergoing a 1° cornering condition are shown in Figure 7. The predicted cornering stiffness from both models is also nearly identical, as listed in Table 2.
Table 2. Full model data for passenger car tire in ABAQUS/Standard.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Standard Mesh</th>
<th>Embedded Mesh</th>
<th>Embedded/Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of elements</td>
<td>25621</td>
<td>19587</td>
<td>0.764</td>
</tr>
<tr>
<td>Number of degrees of freedom</td>
<td>80037</td>
<td>88455</td>
<td>1.105</td>
</tr>
<tr>
<td>Cpu time for steady-state rolling phase (sec)</td>
<td>4745</td>
<td>2553</td>
<td>0.538</td>
</tr>
<tr>
<td>Number of iterations (sdl, equil)</td>
<td>23, 37</td>
<td>23, 31</td>
<td>0.900</td>
</tr>
<tr>
<td>Cpu time (sec) per iteration</td>
<td>79.08</td>
<td>47.28</td>
<td>0.598</td>
</tr>
<tr>
<td>Cornering stiffness (N/deg)</td>
<td>1658</td>
<td>1693</td>
<td>1.021</td>
</tr>
</tbody>
</table>

Figure 6. Contact between the tire and the road surface due to straight rolling. Left figure from standard mesh model; Right figure from embedded mesh model.

Figure 7. Contact between the tire and the road surface due to 1° cornering. Left figure from standard mesh model; Right figure from embedded mesh model.

2.2 Medium radial truck tire

Similar comparisons were made for a medium radial truck tire. This tire has more cord layers and rubber components than the passenger car tire, so more solution efficiency is expected from the embedded mesh technique. The standard mesh and embedded mesh for a typical truck tire are shown in Figures 8 and 9. Again, different shades in the embedded mesh represent different averaged modulus zones. Model sizes are listed in Table 3. In this case, the embedded mesh model has approximately 35% less elements and 16% less degrees of freedom.
Figure 8. Standard mesh of a typical medium radial truck tire.

Figure 9. Embedded mesh model of a typical medium radial truck tire.
Table 3. Half model data for medium radial truck tire in ABAQUS/Standard.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Standard Mesh</th>
<th>Embedded Mesh</th>
<th>Embedded/Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of elements</td>
<td>12230</td>
<td>8102</td>
<td>0.662</td>
</tr>
<tr>
<td>Number of degrees of freedom</td>
<td>44832</td>
<td>37536</td>
<td>0.837</td>
</tr>
<tr>
<td>Cpu time (sec)</td>
<td>2589</td>
<td>689</td>
<td>0.266</td>
</tr>
<tr>
<td>Number of iterations (sdi, equil)</td>
<td>35, 60</td>
<td>20, 27</td>
<td>0.495</td>
</tr>
<tr>
<td>Cpu time (sec) per iteration</td>
<td>27.25</td>
<td>14.66</td>
<td>0.538</td>
</tr>
<tr>
<td>Vertical stiffness (N/mm)</td>
<td>988.3</td>
<td>1002.0</td>
<td>1.014</td>
</tr>
</tbody>
</table>

Rim mounting, inflation, and vertical loading analyses are performed using the half cross-section model. Since steady state rolling is not done in this evaluation, the reflected, full section model is not required. However, for displaying the tire/road contact plots, the full models were run to give a more easily understood picture. The inflated shapes from the two models are overlaid in Figure 10. There is some difference in the results from the two models, especially in the lower sidewall region above the rim. The thickness and stiffness of the tire in this region make the bending response critical for good prediction of the inflated shape. The embedded mesh model, with only two solid elements through the thickness, is overly stiff and so doesn’t adequately predict the vertical stiffness of the tire.

Figure 10. Overlay plot of inflated shapes. Solid line = embedded mesh model; Dashed line = standard mesh model
bending response of the tire about the top of the rim flange. This shortcoming doesn’t
significantly affect the prediction of the model response to vertical loading. The predicted vertical
stiffness listed in Table 3 is nearly identical from the two models. The contact patch between the
tire and ground is shown in Figure 11 for the standard mesh model and the embedded mesh model.
The shapes and pressure distributions are comparable.

These results show that the embedded mesh model can be used to give a much more efficient
solution with nearly identical results for stiffness and tire/road contact. This can be used to
advantage in the tire development process to speed up the analysis phase or to allow more design
iterations to be performed in the same amount of time. The best design for stiffness and contact
response can then be used in subsequent analyses to obtain the detailed stress, strain, and energy
information.

3. ABAQUS/Explicit Applications

An important response to be obtained from a rolling tire analysis is the contact between the tire
and the road surface. All of the forces transmitted from the vehicle to the ground for acceleration,
braking, and steering go through these contact patches. As shown above, the tire/road contact can
be predicted using a steady-state rolling analysis in ABAQUS/Standard if the tread pattern is
ignored. However, the tread pattern can significantly affect the localized contact stresses and
deformations that are important for prediction of performance such as tread wear. In these cases,
the tread pattern is included in the model and ABAQUS/Explicit is used to obtain the solution.
Since the focus of the analysis is on the interface between the tire and the road, and not on the
detailed stress and strain states in the tire cross-section, this is an ideal application for an
embedded mesh. The non-tread portion of the model is only required to provide a sufficient
representation of the structural stiffness response to the applied loading. A more refined mesh can
then be made of the tread portion of the model where the detailed response is needed.

A passenger car tire will be used in the comparison of the standard and embedded mesh models in
Explicit. The tread pattern is not represented in the models for simplicity. However, the behavior
shown by the models and conclusions drawn will be the same as if a tread pattern were modeled.
Standard and embedded meshes for the half cross-section used in the evaluation are shown in
Figure 12. The tire cross-section model has been simplified to remove the portion of the tire that
would lie within the rim. Boundary conditions are applied to the cut face to constrain the tire at
the proper rim width. The half section is reflected to make a full cross-section model for use in the
analysis so that ply steer behavior can be properly represented as the tire rolls.
C3D8R elements are used to model the rubber components and M3D4R elements with attached rebars are used to model the cord layers. A uniform circumferential mesh refinement of 6° is used in both models. The number of elements and degrees of freedom for the two models are listed in Table 4. The embedded mesh gives a 25% reduction in the number of elements and a 1% increase in the number of degrees of freedom.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Standard Mesh</th>
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<th>Embedded/Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of elements</td>
<td>30724</td>
<td>22924</td>
<td>0.746</td>
</tr>
<tr>
<td>Number of degrees of freedom</td>
<td>91656</td>
<td>92556</td>
<td>1.010</td>
</tr>
<tr>
<td>Cpu time</td>
<td>4 hr, 46 min</td>
<td>4 hr, 41 min</td>
<td>0.988</td>
</tr>
<tr>
<td>Number of increments per 2 min</td>
<td>975</td>
<td>1200</td>
<td>1.231</td>
</tr>
<tr>
<td>Cpu interval</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum stable time increment</td>
<td>2.778E-06</td>
<td>2.074E-06</td>
<td>0.747</td>
</tr>
</tbody>
</table>

The analysis is started with the application of boundary conditions to move the tire beads to the proper rim width, followed by application of the inflation pressure. The desired vertical load is then applied. These loadings are applied with smoothed amplitude curves to minimize noise in the solution. The tire model is then accelerated to the desired rolling speed and held at that speed for nearly one revolution. The contact shape and pressure distribution between the tire and the road surface is then extracted for comparison between the two models. Figure 13 shows the contact between the tire and the road surface for the two models at the steady rolling condition. The contact shapes are comparable.
It is expected that the reduction in the number of elements in the embedded mesh model would lead to a corresponding savings in run times, as was seen for the ABAQUS/Standard analyses reported in the preceding section. However, the run times were nearly identical for the standard mesh and embedded mesh models, as listed in Table 4. There are two effects counteracting each other to keep the run times the same - the number of increments solved in a given cpu interval and the minimum stable time increment. These can be found by looking in the .sta files. The values are listed in Table 4 for the two models. As expected, the embedded mesh model solves faster, as measured by the number of increments solved in a given amount of cpu time. For the tire model used in this investigation, the embedded mesh model solves 23% more increments in a 2 minute cpu interval compared with the standard mesh model. However, the minimum stable time increment in the embedded mesh model is 25% smaller than in the detailed mesh model, meaning the solution advances less in each increment. The elements limiting the stable time increment are the same membrane elements in the belt layer of both the standard and embedded mesh. This is not unexpected since the membrane element mesh is the same in the two models and these elements are the smallest and stiffest. However, the added constraints used to embed the membrane elements in the host solid elements reduce the stable time increment. The combination of improved solution efficiency and smaller stable time increment cancel each other in this example, giving no appreciable change in model run time for the embedded mesh model.

4. Summary

The embedded element technique in ABAQUS can be used to advantage in the tire design process when using ABAQUS/Standard. The examples given show that it provides a faster solution with equivalent accuracy for global stiffness and for contact between the tire and the road surface. As such, a tire designer can either shorten the time of the analysis phase of the tire development, or perform more design iterations within the same time span. Some detail in the internal stress and strain states is lost in the embedded element model, especially near the endings of cord layers. If this detail is desired, either a subsequent standard mesh model needs to be run using the preferred design or a combined embedded/standard mesh model can be used for all analyses.

The rolling tire example run in ABAQUS/Explicit showed equivalent tire/road contact results were obtained for both models, but the improvement in solution time was not attained using the embedded mesh model. Thus, there is no compelling reason to recommend one meshing technique over the other for tire analyses run in Explicit.
The ability to perform thermal analysis would be a welcome addition to the capability of the embedded element technique. The prediction of temperature distribution, both during the manufacturing process and during operation on the vehicle or test machine, is of interest to the tire design engineer. These analyses have been successfully performed using a standard mesh model. However, when temperature history and distributions are to be predicted for use in a successive deformation analysis that uses an embedded mesh, the thermal analysis must be done using a standard mesh model. The translation of temperature history from the standard mesh thermal model to the embedded mesh deformation model has been shown to work, but adds a level of complexity to the modeling process.

5. References


6. Acknowledgment

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