Finite Element Prediction of Progressively Formed Conical Stockpiles

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Abstract: Conical piles of granular solids can be found in many industrial sites. These piles are usually progressively formed by depositing from above. A classic question concerning such simple piles is the observation that the pressure distribution beneath the pile shows a marked local minimum beneath the apex which is counter-intuitive as this should be the location expected to have the maximum pressure. Numerous experimental, analytical and computational studies have been conducted to investigate this classical problem over the last few decades, but a comprehensive understanding of the problem remains elusive. A number of recent finite element simulations of the pile have considered the effects of construction history, plasticity and stress-dependence of modulus of the granular solids. Whilst a pressure dip beneath the apex has been predicted, significant uncertainties remain about the effects of these factors on the pressure dip and their interaction.

This paper presents the finite element modelling of a conical stockpile using Abaqus. The effect of construction history was realized by simulating the progressive formation of the conical pile. This was achieved by discretising the final geometry of the stockpile into multiple conical layers and then activating each layer sequentially. The effects of the elastic and plastic parameters were explored. The results show that a pressure dip may or may not be predicted depending on the constitutive model and the values for the model parameters. The study also shows that modelling the conical pile in one single step does not produce the pressure dip. It further shows that the central pressure dip is predicted using a relatively small number of layers and the magnitude of the dip is not sensitive to increasing number of layers, which is in contrast with one previous study.

Keywords: Sandpile, Stockpile, Stress distribution, Pressure dip, Pressure dependent modulus, Progressive layering, Incremental construction.

1. Introduction

The behaviour of granular solids has attracted much attention of researchers from many communities such as Applied Mechanics, Geotechnical Engineering, Chemical Engineering, Materials Handling, Agricultural Engineering and Geophysics. The storage and handling of granular materials is essential to many industries (Nedderman, 1992). Where the material is held
in very large quantities, it is often stored in a stockpile, formed by dumping the solid (e.g. coal and mineral ore) to form a pile whose overall shape is typically conical, but may be prismatic, depending on the method of placement (Figure 1). The solid is often recovered from the stockpile using a conveyor beneath its centre. In such a case, the structure containing the conveyor needs to withstand the pressures exerted by the stockpile. Consequently one key aspect of stockpile design is to determine the pressure pattern beneath it. The experimental finding that there is a significant local reduction in pressure (Figure 2) beneath the apex of the pile below the values one might expect can have a strong impact on the design requirements. This reduction in pressure which is commonly known as the “sandpile” problem has mostly been studied in the past as an interesting scientific anomaly, but the stockpiles make it of considerable economic importance.

**Figure 1. A typical industrial stockpile**

**Figure 2. Vertical base pressure underneath a granular pile**

Despite extensive studies by both the physics and engineering communities over several decades, a comprehensive understanding of the counter-intuitive phenomenon of the pressure dip remains...
elusive. Good reviews of previous analytical, numerical and experimental studies of the problem can be found in Savage (1997, 1998) and Cates et al. (1998). Among these studies, very few adopted the finite element method (FEM), even though it is very powerful and flexible in dealing with complex loading and boundary conditions. This paper investigates the feasibility of adopting the FEM to model the sandpile problem and explores how the observation from such modelling may provide further insight into the physical mechanism of the problem from a continuum point of view.

It has been observed experimentally that the pile construction history is an important factor in the occurrence of central pressure dip beneath the stockpile (Vanel et al., 1999; Geng et al., 2001). For a sandpile formed by distributed deposition (“raining procedure”), no dip was found while for a concentrated deposition (also termed “localized source” or “funnel procedure”), a pronounced dip was observed. It has also been observed that base deflection (e.g. Trollope, 1957; Lee and Herington, 1971) and spatial variation of material stiffness (as introduced in Savage, 1997) can have a significant influence on the extent of the pressure dip. The present study is concerned with the general case that a conical pile is formed on a rigid flat and rough base by concentrated deposition. Finite element simulations of such a general case have been conducted by several researchers. These studies are summarised as below.

Savage (1998) reported results from elastic and elastic-rigid plastic finite element computations for wedges and cones using three different finite element analysis (FEA) packages including Abaqus. They adopted an elastic-rigid plastic Mohr-Coulomb model and modelled the pile using the 8-node quadrilateral element. Many calculations were undertaken to investigate the effects of the internal friction angle $\phi$, dilation angle $\psi$, Poisson’s ratio $\nu$, cohesion $c$ and elastic modulus $E$ of the granular solid. It was found that the results were relatively insensitive to all of these parameters except $\phi$. The predicted vertical base pressure distribution showed no central dip and was almost indistinguishable from the active limit state solution. Simulations using purely elastic, Drucker-Prager and Drucker-Prager/Cap constitutive models produced similar predictions.

Anand and Gu (2000) conducted elastic-plastic calculations of a static conical granular pile with an angle of repose of $\beta=31.5^{\circ}$ using the “double-shearing” constitutive model through a user-material subroutine in Abaqus/EXPLICIT. Two sets of parameters were investigated: one with a constant internal friction angle of $31.5^{\circ}$, the other with a mobilized internal friction angle which evolved from $0^{\circ}$ to $30.0^{\circ}$ with strain hardening. The former produced a state with no plastic deformation and the vertical stress distribution showed a peak under the apex. The latter generated a fully plastic state and the vertical stress distribution showed a pronounced dip under the apex of the conical pile. They concluded that the cause of the dip was the nonhomogeneous plastic strain occurred during the formation of a sand pile, which resulted in a nonuniform internal friction coefficient. The largest plastic shear strain concentrated in a wide inclined band which lies slightly below the pile surface.

Al Hattamleh et al. (2005) also argued that strain localization is the main cause of the pressure dip. In their model, the construction of the granular heap was simulated by incrementally layering in five stages. They adopted a double-slip formulation of “double-shear-type” constitutive model which is similar to that of Anand and Gu (2000) but permits the user to assign the orientations of initial slip lines. Very pronounced stress dips were predicted in all cases except the case of
homogeneous state where the initial slip orientations equal $\pm \pi/4 \pm \phi/2$. They predicted localized vertical plastic strain around the apex with the rest of the pile in an elastic state.

Tejchman and Wu (2008) analysed the pressure distribution under both prismatic and conical sandpiles using a micro-hypoplasticity model which considers the effect of the direction of deformation rate. The construction of the pile was simulated in ten stages with two different methods: horizontal layers and inclined layers, namely the raining procedure and funnel procedure. The results were in qualitative agreement with the experimental results reported by Vanel et al. (Vanel et al., 1999) and the numerical results reported by Al Hattamleh et al. (2005).

Jeong (2005) conducted an extensive FE study on sandpiles using several loading and boundary conditions. In contrast to the other recent studies that adopted complicated constitutive relations, a simple elastic-rigid plastic Mohr-Coulomb model was deployed. A significant effort was invested in developing the “incremental construction” scheme in which the pile was constructed in many stages, which has been used by the silo research community (e.g. Yu, 2004) and Geotechnical Engineering (e.g. Clough and Woodward, 1967; Kerry Rowe and Skinner, 2001). This procedure was shown to be capable of producing a central pressure dip underneath a conical pile. Jeong (2005) also observed that the results are very sensitive to the number of construction layers adopted. A larger number of construction layers predict a bigger pressure dip. The plastic zone in the final stage was predicted to occupy the majority of the pile except the part near the base and the tail ends of the conical surface.

Although some FE simulations of stockpile reported above have successfully predicted a pressure dip, they differ considerably in both the analysis procedure and the final distribution of plastic strain. Some adopted very complicated constitutive models (Anand and Gu, 2000; Al Hattamleh et al., 2005; Tejchman and Wu, 2008), while others used more general elastic-rigid plastic model (Jeong, 2005). Strain localization was argued in two of these studies to be the origin of stress dip (Anand and Gu, 2000; Al Hattamleh et al., 2005), while incremental construction was shown to be a key issue in another two studies (Jeong, 2005; Tejchman and Wu, 2008). There might be some links between these two mechanisms, but they are still unclear.

This study attempts to evaluate the capability of simple constitutive models in predicting the pressure dip in conical stockpiles and to investigate in detail the effect of incremental construction scheme on the pressure profile. The evolution of stress distribution during progressive formation of the stockpile is also shown. The results provide further insight on the potential mechanisms responsible for the pressure dip using such modelling scheme.

2. Reference test data

The stockpile tests conducted by Ooi et al. (2008) with mini iron pellets centrally poured on a rigid base were used as reference data in this study. The pellets are approximately spherical and have a relatively uniform bulk density that is relatively insensitive to packing: the loosest and densest bulk densities achieved in control tests being 2260 and 2370 kg/m$^3$. The internal angle of friction for the pellets was measured to be 34º using a direct shear tester. Five repeat tests were conducted producing a mean pile radius at the base of $R_p = 554mm$ and an average angle of repose of $\beta = 29.0^\circ$. Free-field pressure cells were used to record the normal base pressures underneath the
pile. Figure 2b shows the normal base pressure distribution at the final stage averaged from 5 repeat tests, where a pressure dip in the centre is clearly evident. The average height above the base at different radial positions is also shown.

3. Finite element modelling

3.1 Constitutive models

In this study, relatively simple elastic and elastic-rigid plastic constitutive relations are used in modelling the sandpile. These include linear elasticity (LE), pressure-dependent elasticity (PDE), linear elasticity with Mohr-Coulomb rigid-plasticity (LEMC) and pressure-dependent elasticity with Mohr-Coulomb rigid-plasticity (PDEMC). By adopting simple elastic-plastic models, the role of elastic and plastic parameters in producing the numerical solution can be explored more clearly.

The PDE model used is a Janbu-type relation (Janbu, 1963; Chen and Mizuno, 1990) which is expressed as:

$$\frac{E_t}{P_a} = K\left(\frac{P}{P_a}\right)^m$$  \hspace{1cm} (1)

where $E_t$ is the tangent modulus of elasticity of the granular solids, $P_a$ is the atmospheric pressure (101.3 kPa), $p = -(\sigma_1 + \sigma_2 + \sigma_3)/3$ is the mean pressure, and $K$ and $m$ are experimentally determined parameters. Because such a PDE relation is not readily available in Abaqus, it was implemented as a solution-dependent modulus based on the LE model through the user-subroutine USDFLD. Note that the LE model in Abaqus only accepts the secant modulus. Consequently, Equation 1 is transformed to the following form in terms of the secant modulus and implemented in Abaqus:

$$\frac{E_t}{P_a} = K_s\left(\frac{P}{P_a}\right)^m$$  \hspace{1cm} (2)

where:

$$K_s = K(1 - m)$$  \hspace{1cm} (3)

Because the routine USDFLD provides access to material point quantities only at the start of each numerical time increment, the solution clearly depends on the time increment size or the number of time increment because the material properties remain constant during each increment. Numerical calibration tests were conducted to ensure that this PDE relation was correctly implemented. A frictionless uniaxial compression test with a radius of $r=1.0m$ and a height of $z=1.0m$ was modelled (Fig. 3a). The parameters were chosen as $K_s = 100$ and $m = 0.4$. To avoid numerical difficulties caused by zero elastic modulus at the beginning of the compression, a small
initial elastic modulus of $E_0=0.5\text{MPa}$ was adopted. In this uniaxial compression test, the relation of vertical stress $\sigma_z$ and vertical strain $\varepsilon_z$ for an elastic material can be derived as:

$$\frac{\sigma_z}{\varepsilon_z} = K_s \left( \frac{1 + \nu}{3(1 - \nu)} \right) \left( 1 - \frac{2\nu^2}{1-\nu} \right)$$

(4)

where $\nu$ is the Poisson’s ratio of the solid. The comparison between the input PDE relation and the computation outputs using different number of time (loading) increments $N_t$ is shown in Figure 3b. It is shown that the output curve from the explicit solution approaches the input curve quickly when number of increments increases. Similar convergence tests were also performed for the pile simulations to ensure accurate implementation of the PDE nonlinear elastic treatment.

![Diagram of uniaxial compression test](image)

**Figure 3.** Verification of Janbu elasticity user-subroutine

### 3.2 Problem configuration

Assuming axisymmetry, the conical sandpile was simulated as a triangle in two dimensions. The final sandpile geometry was further discretised into triangular elements using the quadratic 6-node triangular axisymmetric element CAX6. The only load considered is the self-weight of the solids. The bottom boundary of the pile was fixed in both vertical and horizontal directions, representing a rigid and completely rough base. The simulation process was treated as a static problem so the effect of inertia was neglected.
3.3 Incremental construction scheme

The effect of construction history due to the progressive loading of the conical pile was explored by modelling the progressive formation (or incremental construction) of the conical pile. This was achieved by discretising the final geometry of the pile into many conical layers and then activating each layer sequentially, starting from the bottommost layer. This incremental construction process was implemented in Abaqus by using the element removal and reactivation technique through the Model Change keyword. A sketch of the incremental construction with FE mesh arrangements is shown in Figure 4 where the final geometry of pile is divided into several construction layers (denoted by alternative dark and light grey layers). Each construction layer may contain one (Figure 4a) or more layers (Figure 4b) of elements. As a result, the total number of layers of element $N_{el}$ is a multiple of number of construction layers $N_{IC}$. In the present study, the sensitivity of numerical results to both these values ranging from 1 to 60 has been explored.

![Figure 4. Sketch of incremental construction for sandpile](image)

3.4 Input parameters

Unless stated otherwise, the input parameters adopted in all the simulations are listed in Table 1 based on the experimental data described above. The density was chosen as the minimum value from the control tests. This is a reasonable assumption because the particles in a stockpile undergo avalanching during the formation process leading to a relatively loose packing. As suggested by Jeong (2005), the particles are in a state of constant volume condition during avalanches, which corresponds to a Poisson’s ratio of around 0.5. As a result, a large Poisson’s ratio of 0.45 was chosen. The pellets tested were dry and non-cohesive, but a small value of cohesion of 1 Pa is assumed to avoid numerical difficulties.

<table>
<thead>
<tr>
<th>Density $\rho$ (kg/m$^3$)</th>
<th>Angle of repose $\beta$</th>
<th>Angle of internal friction $\phi$</th>
<th>Angle of dilation $\psi$</th>
<th>Cohesion $c$ (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2260</td>
<td>29°</td>
<td>34°</td>
<td>20°</td>
<td>1 Pa</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Elastic modulus $E$ (MPa)</th>
<th>Poisson’s ratio $\nu$</th>
<th>Initial elastic modulus $E_s$ (MPa)</th>
<th>Janbu relation $K_s$</th>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>0.45</td>
<td>0.5 (MPa)</td>
<td>100</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Table 1. Input parameters for sandpile simulations
4. Key results

Two groups of simulations were conducted: one without incremental construction and the other adopting incremental construction. Figures 5a and b show respectively the predicted vertical base pressure distributions underneath the pile for the two groups. The pressure has been normalised by the hydrostatic pressure under the apex at the base \( p = \gamma H \) where \( H \) is the height of the pile. When the whole sandpile was analysed as a single layer (Fig 5a), none of the four constitutive models produced a central dip. This result concurs with the conclusion by Savage (1998). The base pressure is lower than the hydrostatic value towards the centre (at radius \( r = 0 \)) and slightly higher than the hydrostatic value elsewhere, satisfying the global equilibrium in the vertical direction. It is interesting to note that the linear elastic LE model and the non-linear elastic PDE model produced very similar results whilst the two elastic-plastic models LEMC and PDEMC also produced very similar results, but the introduction of plasticity has further increased the shedding of the load away from the centre.

![Figure 5](image.png)

**Figure 5. Vertical base pressure from different constitutive relations \((N_{el} = N_{IC} = 20)\)**

The inclusion of incremental construction produced different effects in each constitutive model, as shown in Figure 5b. The linear elastic-plastic LEMC model produced a shallow dip in pressure whilst the non-linear elastic-plastic PDEMC model produced the most pronounced dip. Both elastic models (LE and PDE) did not predict any dip in pressure. The simulation using incremental construction appears to give rise to a larger shedding of the load away from the centre, thus giving rise to the manifestation of a pressure dip. The results also support the proposition that material
plasticity is a requirement for predicting the sandpile phenomenon and the progressive loading history during sandpile formation also plays a vital role.

![Graph](image)

**Figure 6. Effect of number of construction layers on vertical stress distribution with PDEM (N_{ai}=N_{lc})**

It is noted that the computed base pressure at the centre of the pile on the axis of symmetry shows a larger value that disrupts the smoothness of each curve, except the linear elastic case. Further calculations were carried out exploring different number of construction layers and element layers to investigate the origin of this problem. It was observed that this larger pressure at the central node is always confined to only within the first column of elements next to the central axis. The source of the problem may thus be associated with the computation of the nodal stress in the elements containing nodes where zero radial displacement is imposed and stepwise incremental construction scheme is used. This issue is further illustrated in Figure 6a which shows five calculations with increasing number of construction layers. The nodal pressure at the centre was always predicted to be larger no matter how many incremental layers were used. This issue is under further investigation. Figure 6b shows the same results as Fig. 6a with the central point of data omitted. It is evident that this very local occurrence does not influence the overall prediction, so in the rest of this paper, the central point will not be plotted. It should be noted that no previous FEA studies have reported this phenomenon.

Figure 6b reveals that the shape of predicted pressure distribution and size of central pressure dip are not sensitive to the number of construction layers in the range tested in this study (N_{lc} = 5 to 60). This is contradictory to the observation of Jeong (2005) where the size of the dip increases with an increasing number of construction layers. In another independent study of progressive filling silo wall pressure using a similar incremental scheme, Yu (2004) concluded that the
calculated peak wall pressure converged quickly and was not sensitive to the number of filling layers used, which is in agreement with the observation here.

Since the nonlinear elastic-plastic PDEMC model produced the best prediction, the rest of this paper will focus on the results using the PDEMC constitutive model with incremental construction. Figures 7-9 show the FEA results using 40 layers of elements and 40 construction layers \((N_{el}=N_{el}=40)\). Figure 7a shows the vertical stress along horizontal paths at different heights in the pile at the final stage of construction. The FEA predicted that the central dip in vertical stress also exists within the pile but it reduces quickly from the base upwards. This conclusion is consistent to that drawn by Anand and Gu (2000). Figure 7b shows the evolution of the normal base pressure during the incremental simulation process. It is clear that the pressure dip is experienced throughout the pile formation process from the very beginning.

The contours of the vertical stress \(\sigma_v\) and the mean pressure \(p = -(\sigma_1 + \sigma_2 + \sigma_3)/3\) for the whole pile are shown in Figures 8a and b respectively. Because the tangent elastic modulus is dependent on the mean pressure according to Eq. 1, the variation of the elastic stiffness in this model is directly related to the mean pressure. The stiffness is predicted to be increasing with depth as one would expect. More importantly, the largest stiffness at each level is some radial distance away from the centre, giving rise to a softer central core surrounded by stiffer surrounding regions. The analysis has thus identified an arching mechanism arising from the stress dependency of the bulk stiffness in which the vertical load is attracted to the stiffer zone, away from the softer zone.

**Figure 7.** Vertical stress along horizontal paths using PDEMC elastic-plastic model
Figure 8. Stress distribution in pile (Pa)

Figure 9. Comparison between FEM predictions and test results

The predicted normal base pressure distribution is compared with the experimental result in Figure 9. The FEA predicted a smaller dip than observed in the experiments. One of the possible causes for this discrepancy is that the test piles were slightly rounded at the top due to the impact of the pouring pellets whilst a perfect conical pile was assumed in the numerical simulations, resulting in a smaller apex height in the actual pile than in the numerical simulation. It is also possible that a better match can be produced by varying some of the input parameters including the dilation angle, the Poisson’s ratio and the Janbu elastic parameters. Further parametric investigation is being undertaken and will be reported elsewhere.
5. Conclusion

A finite element analysis of a conical sandpile has been presented and compared with experimental observations. The key aspects of modelling a sandpile using the finite element method have been discussed and the outcomes for several elastic and elastic-plastic models, with and without incremental layer construction scheme, have been presented. The chief conclusions of the study are:

1. Incorporating plasticity and simulating the progressive construction of the sandpile are both necessary for the finite element method to predict the classic sandpile pressure distribution where a significant dip exists beneath the apex of the pile.

2. Whilst the FE calculations have produced a reasonable prediction of the pressure distribution, they under predict the size of the central dip. A closer match may be achieved by modelling the actual shape of the test pile and adjusting the input parameters.

3. The size of the pressure dip and the overall pressure profile are found to be insensitive to the number of construction layers used. This contrasts with the observation of Jeong (2004) where the dip was reported to become larger as the number of layers increases.

4. The largest stiffness was predicted to be some radial distance away from the centre, giving rise to a softer central core surrounded by stiffer surrounding regions. The analysis has thus identified an arching mechanism arising from the stress dependency of the material stiffness in which the vertical load is attracted to the stiffer zones, away from the softer central zone.

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7. References

