The Influence of Friction-Induced Damping and Nonlinear Effects on Brake Squeal Analysis

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ABSTRACT

This paper presents numerical studies of friction-induced instabilities of brake systems using complex mode analysis. The complex eigenvalue extraction is performed at a deformed configuration; thus, nonlinear effects are taken into account in the modal analysis. An example case is used to illustrate the importance of friction-induced damping and nonlinear effects in brake squeal analysis. It is found that the inclusions of lining wear, geometric nonlinearities, and positive as well as negative frictional damping effects have significant influence on the brake squeal predictions.

INTRODUCTION

Brake noise is widely accepted as a friction-induced dynamic instability. There are two main categories of numerical methods that are used to simulate this phenomenon: transient analysis and complex mode analysis. Because the transient method is computationally expensive, many scholars and engineers use complex eigenvalue analysis to study friction-induced dynamic instabilities [1–9].

Although the complex mode analysis was successfully used in brake squeal problems, it still cannot be considered a predictive tool. The drawbacks of this method are over-predictions and missing, unstable modes in the squeal frequency range.

User experience and engineering judgment are essential to obtain reliable results using the complex mode analysis. This includes the modeling of boundary conditions, the choice of element connections, and the inclusion of brake components. An example of the inclusion of brake components can be seen in [8], where Shi et al. include insulators in the model to better predict the damping effect. In another example, Kung et al. [9] compare the predictions of low-frequency drum brake squeal between models with and without the suspension components. It was found that both models predict the unstable frequency, but the model without suspension was not able to reflect a design modification.

Another direction for improving the complex eigenvalue predictability is the analysis tool. Unstable modes can be calculated using general-purpose finite element software by adding user-defined friction coupling. Dynamic instability can also be obtained by using specialized software that is dedicated to the brake squeal analysis. However, the accuracy of results depends on the formulation of friction coupling, the algorithm for complex eigenvalue extraction, as well as the inclusion of damping, nonlinearity, and lining wear effects.

In recent years many efforts have been made to increase the predictability of the complex eigenvalue method. Lee et al. [4] implemented the MacNeal method in their analytical model. Hamzeiz et al. [5] and Moir et al. [6] took into account friction-induced positive damping effects. Bajer et al. [1] and Kung et al. [7] used the new feature in ABAQUS Version 6.4 to simulate brake squeal by combining the nonlinear static analysis with complex eigenvalue analysis.

This paper continues the work of [1] and [6] with a focus on the evaluation of the features in ABAQUS Version 6.4, including friction-induced damping and nonlinear effects. In addition, a new feature in ABAQUS Version 6.5 that takes lining wear effect into account is also used to better model the contact condition. A front disc brake with high frequency squeal modes is used as the example, and dynamometer test results are used for correlations. The dynamometer test of the braking process is treated as a quasi-static process, and the nonlinear static analysis of the system in motion is performed first to identify static equilibrium under applied loads. The tests are followed by the complex eigenvalue extraction procedure, which is conducted to identify possible system instabilities. The combination of nonlinear static analysis and complex eigenvalue
extraction allows investigation of the influence of nonlinear effects and the lining wear effect.

FORMULATION

FRICITION CONTRIBUTION

According to the Coulomb law the friction contribution to the virtual work can be expressed in the following form:

$$
\delta W = \int_A \tau_i \delta \gamma_i \, dA,
$$

where

- $A$ - area of contact,
- $\delta \gamma_i$ - virtual relative slip,
- $\tau_i = \mu(\gamma, p) n_i$ - tangential stress,
- $\mu = \mu(\gamma, p)$ - friction coefficient,
- $p$ - contact pressure,
- $\gamma = \sqrt{\gamma_1^2 + \gamma_2^2}$ - equivalent slip rate, and
- $n_i = \frac{\gamma_i}{\gamma}$, $i = 1, 2$ - normalized slip direction.

Next, the perturbation of virtual work is

$$
d \delta W = \int_A d \tau_i \delta \gamma_i \, dA = d \delta W^K + d \delta W^{D_+} + d \delta W^{D_-}.
$$

The first term in the equation above,

$$
d \delta W^K = \int_A \left( \mu + \frac{\partial \mu}{\partial p} p \right) n_i \delta \gamma_i \, dp \, dA,
$$

yields the unsymmetric contribution to the stiffness matrix. This term is essential to capture the phenomenon of mode coupling.

The second term,

$$
d \delta W^{D_+} = \int_A \frac{\partial \mu}{\partial \gamma} \gamma_i n_j \delta \gamma_i \, d\gamma_j \, dA,
$$

yields the contribution to the damping matrix and exists only if friction coefficient depends on the relative slip rate (velocity). In the case of negative friction-velocity gradient (the usual case) this contribution is negative semi-definite and can trigger instabilities. This term couples the frictional stresses to the tangential vibrations of the pads along the slip direction.

Finally, the last term,

$$
d \delta W^{D_-} = \int_A \frac{\mu p}{\gamma} (\delta_{ij} - n_i n_j) \delta \gamma_i \, d\gamma_j \, dA,
$$

yields the symmetric contribution to the damping matrix and is associated with the friction forces stabilizing the vibrations along the contact surface in the direction perpendicular to the slip direction (radial direction). This term is proportional to the contact pressure but is inversely proportional to the slip velocity. It couples the frictional stresses to the radial vibrations of the pads. Taking this term into account may suppress some false unstable modes, which could be reported by the complex mode analysis without taking this term into account.

COMPLEX MODE ANALYSIS

The complex eigenproblem is solved using the subspace projection method; thus, the natural frequency extraction analysis must be performed first to determine the projection subspace. The complex eigenvalue problem can be stated as follows:

$$
(\lambda^2 [M] + \lambda [C] + [K]) \{\Phi\} = 0,
$$

where $[M]$ is the mass matrix, which is symmetric and positive definite; $[C]$ is the damping matrix, which can include friction-induced damping effects as well as material damping contribution; $[K]$ is the unsymmetric (due to friction contributions) stiffness matrix; $\lambda$ is the eigenvalue; and $\{\Phi\}$ is the eigenvector. Because the eigenvalue extraction is performed at a deformed configuration, the stiffness matrix, $[K]$, can include initial stress and load stiffness effects. Both eigenvalues and eigenvectors may be complex. This system is symmetrized by dropping the damping matrix $[C]$ and unsymmetric contributions to the stiffness matrix $[K]$. In this case $\lambda$ becomes a pure imaginary eigenvalue, $\lambda = i \omega$, and the eigenproblem can be written as follows:

$$
(-\omega^2 [M] + [K_s]) \{\phi\} = 0.
$$

This symmetric eigenvalue problem is solved using the Lanczos or subspace iteration eigensolver. Next, the original matrices are projected onto the subspace of real eigenvectors. The reduced complex eigenvalue problem is solved using the QZ method for generalized, nonsymmetric eigenproblems with dense matrices. Finally, the eigenvectors of the original system are recovered. For more detailed description of the formulation and the algorithm we refer to [1].
CASE STUDY

Numerical simulations were performed using the model of a front disc brake system, which consists of a rotor, caliper, bracket, piston, and pads. A finite element mesh with 26,000 elements and 825,000 degrees of freedom is shown in Figure 1. Surface-based contact is defined between the rotor and the pads. Dynamometer tests results for this brake system (Figure 2) show major squeal modes at about 6, 7, 10, 13, and 16 kHz. The predictions, obtained using the complex eigenmode analysis, are compared to the dynamometer tests.

In the first step of the brake squeal analysis the contact between the linings and the rotor is established by applying pressure of 1.38 MPa. Next, the slipping condition on the lining-rotor interface is defined by imposing a rotational velocity of 2.5 rad/s on the rotor. The friction coefficient depends linearly on velocity with the friction-velocity gradient set to $-3 \times 10^{-6}$ sec/mm. The average value of $\mu$ is 0.4.

INFLUENCE OF FRICTION-INDUCED DAMPING

To study the influence of friction-induced damping effects on the prediction of instabilities, we perform a sequence of three complex mode analyses. In the first analysis none of the friction-induced damping effects is taken into account—only the unsymmetric contribution to the stiffness matrix due to friction is included. Figure 3 presents real parts of complex eigenvalues (squeal propensity) as a function of frequency. Although many of unstable modes are predicted correctly, a major squeal mode at about 16 kHz is missed.

In Figure 4 shows the results of the complex mode analysis with unsymmetric contribution to the stiffness matrix due to friction and negative damping effects. The major squeal mode at 16 kHz is predicted correctly, but many overpredictions are reported—a major drawback of the complex mode analysis. The 16 kHz squeal mode is a shear mode—a tangential mode with the rotor cheeks moving out-of-phase. As it is presented in [2], tangential modes are more sensitive to the negative $\mu - v$ slope; thus, it is not surprising that taking into account the friction-induced negative damping effect allows for more accurate prediction this type of squeal mode.
Finally, the complex mode analysis is repeated to reduce the number of overpredictions; this time both positive and negative friction-induced damping effects are included in addition to unsymmetric contribution to the stiffness matrix. As mentioned above, the positive damping is not an artificial term that is added to reduce overpredictions. This term occurs in the variational formulation when friction-induced contributions are derived for sliding contact. Thus, excluding this term in complex mode analysis introduces additional approximation. As expected, the number of overpredictions is reduced significantly, but the unstable tangential mode at 16 kHz becomes more prominent.

To provide more insight into the effects of friction-induced damping, we present the influence of the negative and positive damping on the unstable modes at 6 and 13 kHz. The 6 kHz squeal is dominated by an out-of-plane mode. The 13 kHz squeal is the combination of the first shear mode and tenth nodal diameter mode of the rotor. It also includes radial motion of linings. The squeal propensity at 6 kHz for the analysis without friction-induced damping effects is 278. If positive friction damping is taken into account, the squeal propensity is decreased by 23%. On the other hand, the negative friction damping increases the real part of the eigenvalue by 5%. For the squeal mode at 13 kHz, the real part of the eigenvalue is 96 if no friction-induced effects are taken into account. The positive damping decreases the squeal propensity by 80% and the negative damping increases it by 20%. Because the 13 kHz squeal is a tangential mode, it is sensitive to the negative friction damping. This squeal mode also includes radial motion of pads what makes the positive friction-induced damping effects prominent.

MODELING OF LINING WEAR

It is known that lining wear affects contact condition and noise generation. In a dynamometer test or vehicle test, brake linings have to be burnished before they can be used for testing. Similarly, in a finite element analysis using an unworn lining profile can generate unrealistic contact pattern that causes discrepancies in noise predictions.

During the early years of complex eigenvalue analysis uniform pressure distribution was assumed and lining wear considered. As the technique evolved, friction coupling was formulated based on the contact conditions; therefore, the lining wear effect became more important. Unfortunately, even though the worn lining profile can be measured, it is difficult to incorporate it in the model. In this paper lining wear is calculated to obtain a more realistic contact pattern that improves complex eigenvalue predictions.

Up to this point the analysis has been using the unworn lining profile. Figure 6 presents the contact pressure distribution on both inboard and outboard linings after the pressure is applied and the sliding condition between the rotor and pads is imposed. The dark blue color indicates zero contact pressure—this part of the lining is not in contact. The contact pressure is concentrated in the upper part of both linings with some bias toward the
leading edge. A significant part of the pads is not in contact, which is not realistic.

Figure 6. Contact pressure distribution on the inboard (above) and outboard (below) linings without wear effects.

To obtain more realistic contact pressure distribution, we take into account the effect of linings wear. The wear is modeled by moving the nodes on the lining surface in the normal direction. It is equivalent to removing some material on the contact surface of pads. A simple wear function is used:

\[ w = kp, \]

where \( w \) is the distance by which a node is moved in the normal direction, \( p \) is the contact pressure, and \( k \) is a constant. The wear is applied incrementally; and after every increment, equilibrium iterations are performed to ensure that the system is still in equilibrium after some material is removed. In this analysis the nodes are moved very little—less than 0.005 mm—but the difference in the contact distribution is very significant (Figure 7). The area of lining that is not in contact is reduced, and the contact pressure is distributed more evenly. Even better pressure distribution can be obtained if more sophisticated wear function is employed or more material is removed.

Next, the complex mode analysis is performed at the configuration at which wear effects are taken into account. Figure 8 shows squeal propensity (a real part of a complex eigenvalue) as a function of frequency.

All the squeal modes are predicted correctly. Even the unstable tangential mode at 16 kHz, for which it was necessary to include negative damping, is found when the wear effects are included. For a more detailed description of the algorithm used to model wear, see ABAQUS documentation [7, 8].

Figure 7. Contact pressure distribution on the inboard (above) and outboard (below) linings with the wear effects taken into account.

Figure 8. Squeal propensity as a function of frequency.
EFFECT OF GEOMETRIC NONLINEARITIES

To demonstrate the effects of geometric nonlinearities, we compare the natural frequencies computed at the undeformed and deformed configuration. First, a small amount of pressure is applied to establish contact and the natural frequencies are extracted. These frequencies are considered as obtained at the initial (undeformed) configuration. Next, the higher pressure (2.74 MPa) is applied and the natural frequencies are extracted again. In this analysis no change of contact conditions is allowed (the same nodes stay in contact) when additional pressure is applied to make sure that the change of contact constraints does not affect the results. Figure 9 shows the differences in natural frequencies between the deformed and undeformed configuration. For some frequencies the difference is up to 14%. The effect of geometric nonlinearities on the complex mode analysis is even more pronounced. Thus, performing a brake squeal analysis at the undeformed configuration may lead to less accurate results.

Figure 8. Squeal propensity for the analysis with wear effects taken into account.

Figure 9. Differences in natural frequencies between the deformed and undeformed configuration.

Usually, when the applied pressure is increased, it causes changes in contact constraints. If this effect is taken into account, the differences between frequencies at the deformed and undeformed configuration are up to 23%.

CONCLUSION

This paper uses a front disc brake system to demonstrate the effects of various new techniques used to enhance brake squeal prediction. First of all, the effect of lining wear is included in the analysis to obtain more realistic contact patterns. Complex eigenvalue results are shown to be more accurate as lining wear is introduced.

Secondly, the effects of positive and negative friction-induced damping are presented. The inclusion of negative damping due to velocity dependent friction coefficient increases the instability of a squeal mode that has large tangential motion. The activation of positive friction-induced damping reduces over-predictions and makes actual squeal modes more prominent.

Finally, it is shown that including geometric nonlinearities in the frequency extraction analysis of brake systems may affect the results significantly. The implementation of these new features has made the complex eigenvalue analysis a more accurate tool for solving brake squeal problems.
REFERENCES


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