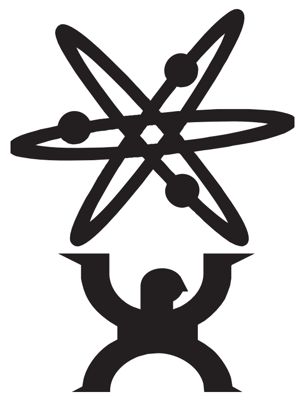
Bodie Technology, Inc
Smart-Tools for Analysis™

Using Advanced Energy Methods to Enhance FEA and Experiments

Ted Diehl, PhD

Kinetic Energy
Viscous Dissipation
Potential Energy
Internal Energy
Artificial Energy
Frictional Dissipation
Plastic Dissipation
Energy Derivatives



Overview

- Conservation of energy and the energy balance
- Typical FEA analyst's use of energy
  - Assessing energy quantities to determine plausibility of FEA
- Advanced uses of energy and energy methods
  - Using Energy Methods to estimate impact behavior from quasi-static data
    - Nonlinear and linear problems
  - Causality and Energy Derivatives
  - Using Derivatives of Energy to
    - Correct/adjust models to remove undesirable distortions
    - Create fast predictions from existing data

Several examples demonstrating techniques that enhance Abaqus FEA and experiments

Conservation of Energy and the Energy Balance

Abaqus outputs all these energy quantities, and more, if you ask for it!

Applied External Work =

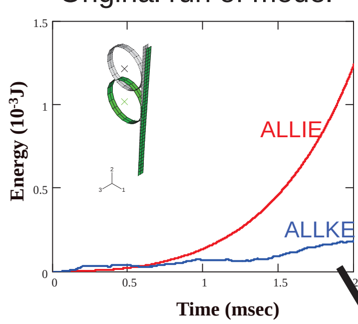
Internal Strain Energy + Kinetic Energy + Dissipation Effects

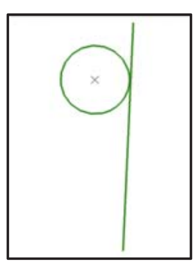
- **WK + HF = IE + KE + FD + VD + IHE – Other**
  - **IE = SE + PD + CD + DMD + FC + AE + DC**
- **Various energy terms**
  - External Work, External Heat Flux, Internal Strain Energy, Kinetic Energy, Frictional Dissipation, Viscous Dissipation , Internal Heating, Energy associated with Penalties and Mass Scaling, Recoverable Strain Energy, Plastic Dissipation, Viscoelasticity, Damage, Fluid Cavity Energy, Artificial Hourglass Control Energy, Distortion Control, and the Kitchen Sink!

Typical Use of Energy Quantities to Assess FEA Plausibility

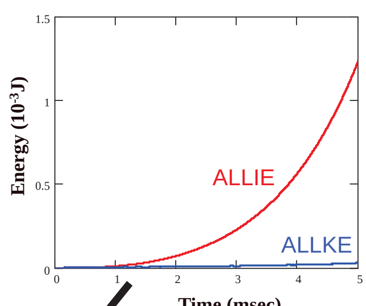
- Energy plots are helpful in Explicit Dynamics models to determine if Kinetic Energy is significant.
  - This is very helpful when performing quasi-static analysis via Explicit FEA.

Original run of model

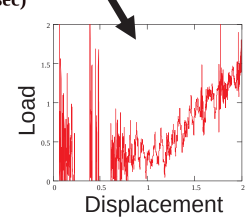


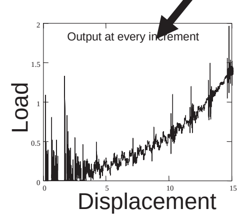


Model run with 2.5x slower loading



Model run too fast for quasi-static





Good quasi-static result, DSP can cleanup the rest

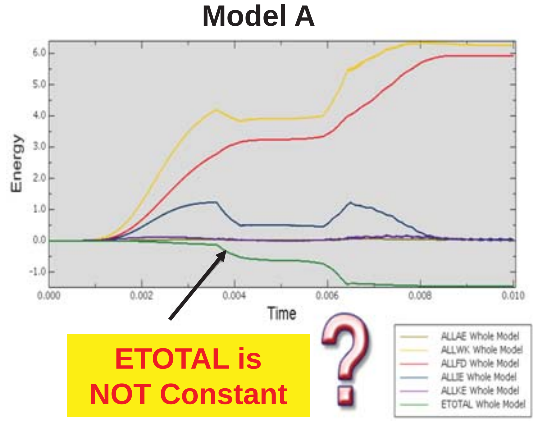
Checking the Energy Balance with ETOTAL

- In Abaqus, the energy balance equation for the entire model is written as
  - **ETOTAL = Σ_{ALL}(- WK - HF + IE + KE + FD + VD + IHE – Other)**
    - Same equation as before, but the work is moved to same side as other quantities and ETOTAL is introduced.
    - ETOTAL is a catch-all quantity.
    - **ETOTAL should remain constant throughout the entire solution.**
      - It is typically zero, or very near zero.
      - Models with initial kinetic energy will have large non-zero ETOTAL, but it should still be constant over the solution duration.

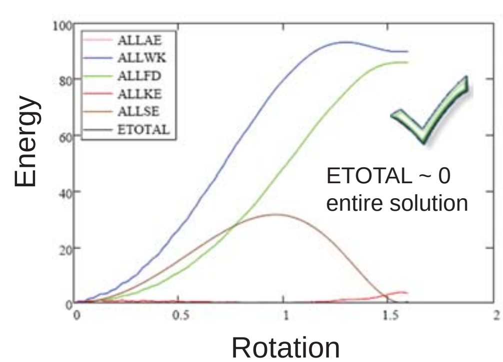
Using an ETOTAL Check to Find a Problem with a Model

- Here are energy assessments from two unrelated models.
  - **Model A** likely has something drastically wrong, as ETOTAL varies significantly relative to other energy terms.
    - **This plot led the analyst to ultimately find a bug in the model.**
  - **Model B** indicates that ETOTAL is constant and thus the results pass this sanity check.

Model A

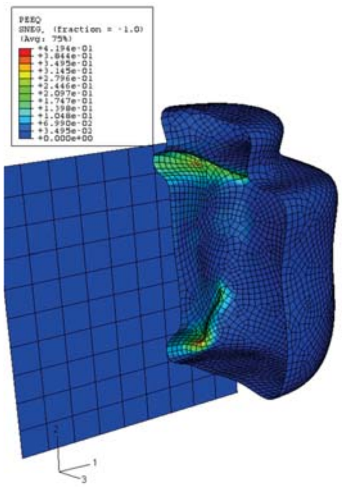


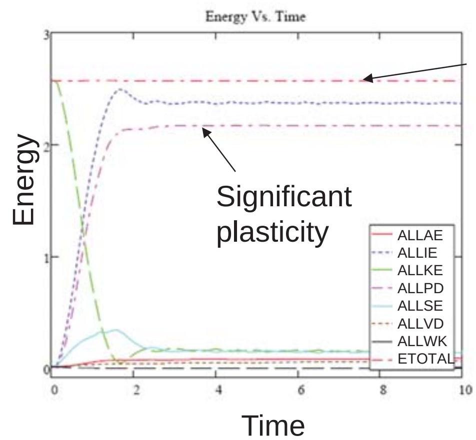
Model B



Unfortunately, Energy is NOT in the front of FEA Users Minds

- Most FEA analysts look at deformed shapes, stress and strain contours, displacements and reaction forces.
  - **Many less FEA users frequently look at plots of energy**
    - **Very few do any additional calculations with energy quantities**



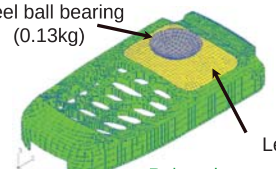


ETOTAL is non-zero, BUT constant. It is non-zero due to large initial kinetic energy in the problem.

Using Energy Methods to Estimate Impact Behavior from Quasi-Static Data

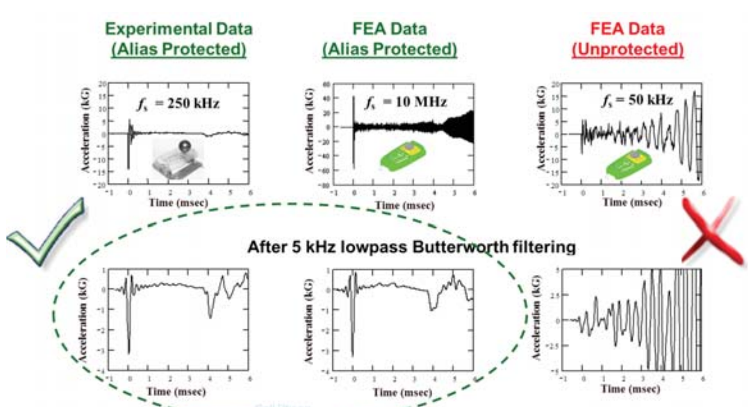
Accelerometer mounted bottom-side of lens

Steel ball bearing (0.13kg)



Lens

Polycarbonate housing



Can I do any useful analysis of a severe impact problem like this without resorting to a transient dynamic analysis ???

Ref: Diehl, et. al., "Using Digital Signal Processing …", *Abaqus Users' Conference,* Chester, United Kingdom, 1999.

Energy Methods to Estimate Impact Behavior

- **Basic approach and assumptions**
  - Combine Static Analysis & Energy Methods to predict Impact results.
    - Energy is always conserved (underlying principle).
  - **Kinetic Energy** (just before impact) ⇒ **(IE + FD)** at "peak event time"
    - IE contains elastic and inelastic energy.
  - **Apply Static Load/BCs until IE + FD = KE** (prior to impact)
    - Statically deform structure into the expected dynamic deformation.
  - The energy transfer assumption is typically a conservative assumption and only good for certain cases.
    - In a general drop problem, the structure will always have a mix of kinetic and stored internal energy throughout the impact event.
  - Considers only "1st mode deformation".
    - Higher modes are ignored.
  - Material strain rate sensitivity is ignored.
    - Use properties "at rate" in quasi-static simulation (not possible in physical test)

Energy Methods to Estimate Impact Behavior

- **Additional comments**
  - Must define sufficient static displacement B.C.'s to avoid rigid body motion.
    - Consider the use of Inertial Relief in certain simulations to avoid over constraining with displacement BCs.
  - Understanding what is the correct static deformation mode to apply is not too difficult in some cases & nearly impossible in other cases.

- **Accuracy**
  - Most accurate for Displacement results.
  - Stress and Strain are reasonably accurate.
  - Acceleration not computed in a static model.
  - Strain-Rate Dependence of Material Behavior is Ignored.
    - May be significant in plastics and polymers!

Remember, this is only an approximation and the underlying assumptions must be valid.

Energy Methods to Estimate Impact Behavior

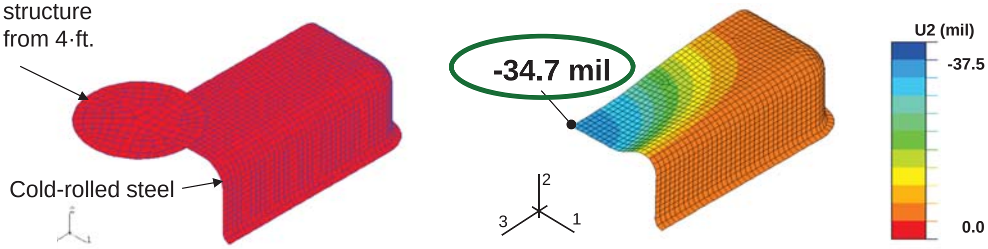
- **Steel Shell Denting**

Explicit Dynamics Model That Includes Plasticity

Rigid mass impacts structure after fall from 4·ft.

Undeformed Shape

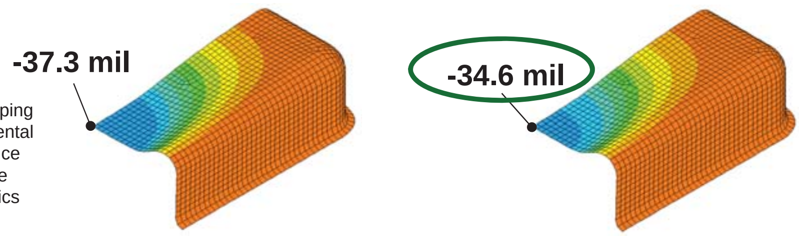
Rigid body impacted structure, then bounced off, and finally the structure has undergone an elastic spring-back



Static Model That Includes Plasticity

Deformed shape when IE equals KE of rigid body prior to impact

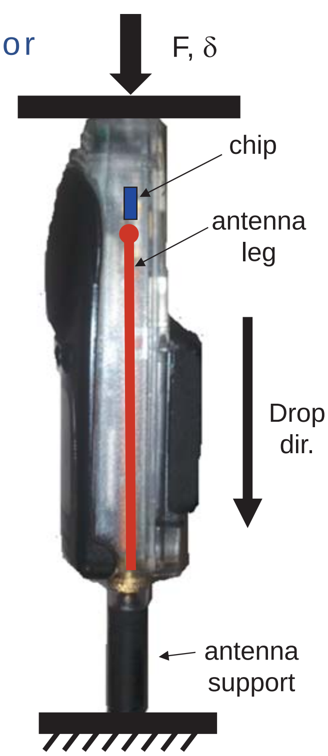
Deformed shape after rigid body is removed and elastic spring-back has occurred



Ref: Diehl, T., "Developing Simulation & Experimental Methods ...," Conference on Plastics for Portable and Wireless Electronics …, October, 2000.

Energy Methods to Estimate Impact Behavior

- **Phone drop problem**
  - Design a physical test to determine if the antenna support will compress enough to have the antenna leg deform too far into the phone housing, causing a chip on the circuit board to be broken off?
    - Phone is dropped from 2·m
- **Solution**
  - Assume that housing stiffness is much larger than antenna support and chip/board stiffness
  - Compress phone in Instron (or MTS) uniaxial tester.
  - Record load and cross-head displacement
    - Integrate to get applied work vs displacement
  - Stop test when work equals mgh
    - M = mass of phone, $g = 9.81 \cdot m/s^2$, $h = 2 \cdot m$



Ref: Diehl, T., "Developing Simulation & Experimental Methods ...," Conf. on Plastics for Portable and Wireless Electronics …, October, 2000.

Simple Scaling Equation to Estimate Impact Behavior for Problems with Linear Response

- Linear Structure - No plasticity or changing contact.

$$F = Ku \qquad SE = \frac{1}{2}Ku^2 \qquad W = \int_0^x F(u)\,du = \frac{1}{2}Fx$$

- Energy from Dynamics (Potential Energy & Kinetic Energy)

$$PE = mGh \qquad KE = \frac{1}{2}mv^2$$ m = mass, h = drop height, v = velocity

  - These energies determine **DEPTI** (Dynamic Energy Prior To Impact).

- Scale any static response from model, r_{static}, to get dynamic response, r_{dyn}, by

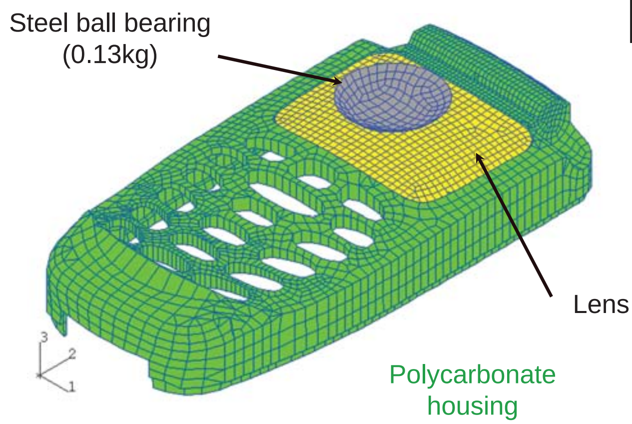
$$\boxed{r_{dyn} = r_{static}\sqrt{\frac{\text{DEPTI}}{SE + FD}}}$$

Structure is statically deformed into expected "impact deformation mode"

Simple Scaling Equation to Estimate Impact Behavior for Problems with Linear Response

- **Lens Impact Analysis**
  - **Steel ball bearing dropped 0.5m**

$$\boxed{r_{dyn} = r_{static}\sqrt{\frac{\text{DEPTI}}{SE + FD}}}$$



Max Lens Displacement

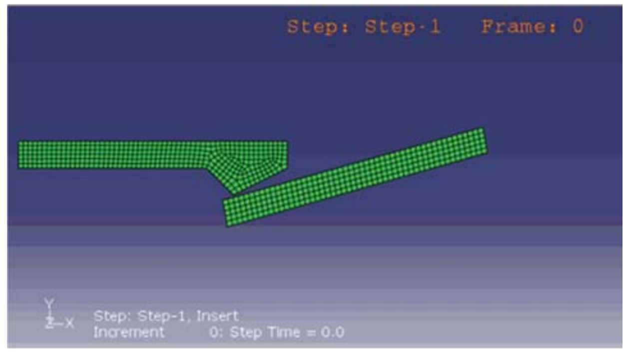
Estimate via Statics + Energy Meth.	Actual Impact Simulation
3.68mm	**3.76mm**

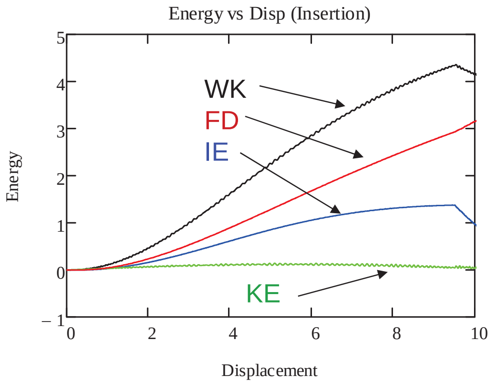
Scaled static estimate accuracy 0.979

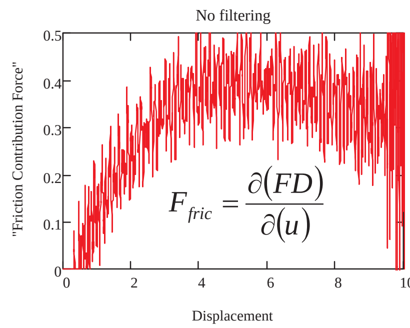
Quasi-static solution derived from model published in the following paper:
Diehl, et. al., "Using Digital Signal Processing …",
Abaqus Users' Conference, Chester, United Kingdom, 1999.

Causality and Energy Derivatives

- Can Energy tell me what's the largest **<u>Cause</u>** of the force I feel when I load a structure?
  - Corollary – What materials, components, etc. have little influence on performance?







Causality and Energy Derivatives

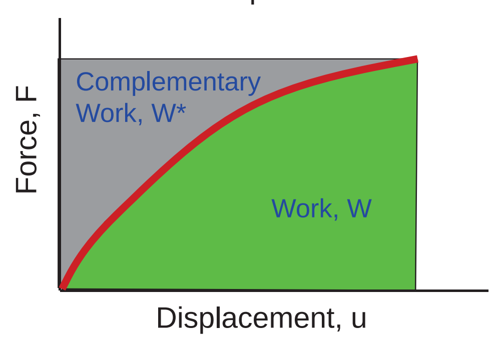
- Basic relationships
  - External Work

$$W = \int F\,du$$

  - External Complementary Work

$$W^* = \int u\,dF$$

Measures of work from applied loads and displacements

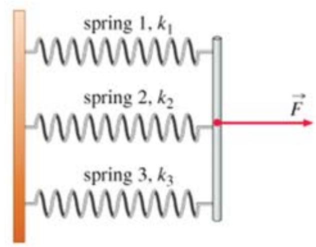


  - **Observations**
    - $$\frac{\partial(WK)}{\partial(u)} = F_{applied}$$
    - Derivatives of external complementary work with respect to forces produce displacements or displacement-like quantities.
      - Derivative with respect to rotation produce moments.

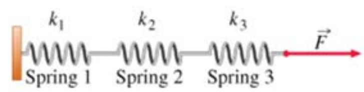
Causality and Energy Derivatives

- Simplified Energy Balance (ignoring some terms)
  - **WK = IE + KE + FD**
- Derivatives of energy relate directly to the force balance for springs in parallel

$$\frac{\partial(WK)}{\partial(u)} = F_{applied} = \frac{\partial(IE)}{\partial(u)} + \frac{\partial(KE)}{\partial(u)} + \frac{\partial(FD)}{\partial(u)} = \Phi_{IE} + \Phi_{KE} + \Phi_{FD}$$

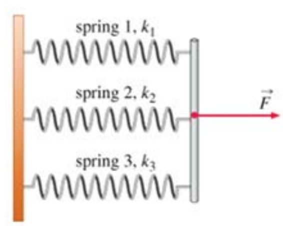


- **Springs in Parallel**
  - Force balance: $F_{total} = F_1 + F_2 + \ldots$
  - Displacements: $\Delta L_{total} = \Delta L_1 = \Delta L_2 = \ldots$
- **Springs in Series**
  - Force balance: $F_{total} = F_1 = F_2 = \ldots$
  - Displacements: $\Delta L_{total} = \Delta L_1 + \Delta L_2 + \ldots$



Need force derivatives of Complementary Work

Causality and Energy Derivatives



- Simplified Energy Balance (ignoring some terms)

  WK = IE + KE + FD
- Derivatives of energy relate directly to the force balance for springs in parallel

$$\frac{\partial(WK)}{\partial(u)} = F_{applied} = \frac{\partial(IE)}{\partial(u)} + \frac{\partial(KE)}{\partial(u)} + \frac{\partial(FD)}{\partial(u)} = \Phi_{IE} + \Phi_{KE} + \Phi_{FD}$$

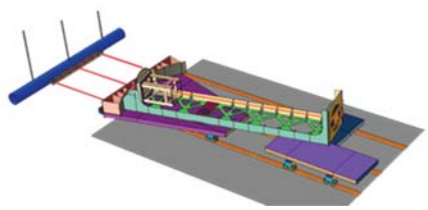
- **Causality Coefficients** $\Rightarrow \Phi_{IE}, \Phi_{KE}, \Phi_{FD}$
  - Terms assess contributions to overall structural response.
  - Use to determine what to change to have most influence on response.
    - Can be grouped by part, material, element set, …
  - Can make estimated prediction equations for changes in quantities without the need to run the entire model again (or run a physical test again)!

Causality and Energy Derivatives – Creating a "Hit List"

- Given a structure with a variety of components and materials, how can a "Hit List" of best candidates for modification be created to increase (or decrease) overall structure stiffness?
  - Can this be done with single simulation run?
  - Will it work for nonlinear problems too?

- **Answer – YES**
  - **Via Causality and Energy Derivatives**

- Analyze model for a specified set of BC's and loads (linear or nonlinear).
  - Output IE (and other energies) grouped by component, material …
  - Derivatives of energy relative to applied BC displacement
    - **⇒ Causality Coefficients**



WOX Shock Tester with permission from Jon Yagla, Naval Surface Warfare Center, Dahlgren Div.

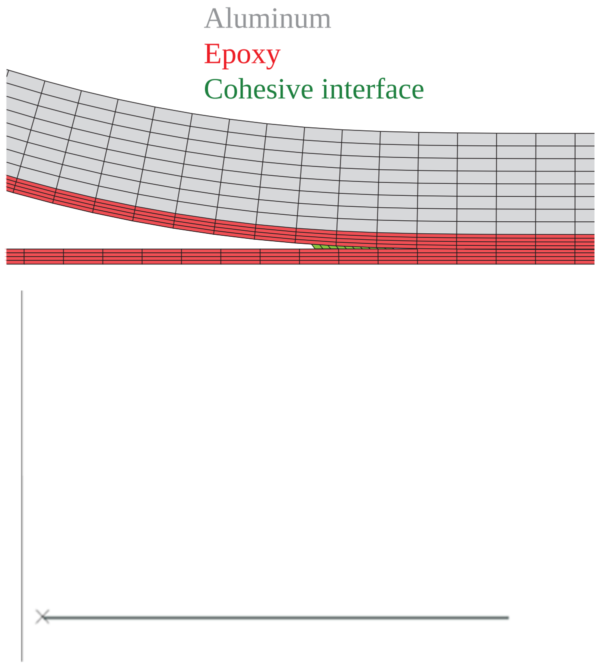
Causality and Energy Derivatives

$$\frac{\partial(WK)}{\partial(u)} = F_{applied} = \frac{\partial(IE)}{\partial(u)} + \frac{\partial(KE)}{\partial(u)} + \frac{\partial(FD)}{\partial(u)} = \Phi_{IE} + \Phi_{KE} + \Phi_{FD}$$

- Estimating changes in a structures response from energy derivatives
  - Derivatives of **IE** ⇒ how component stiffness changes affect response.
    - Stiffness changes including changes in material or geometry.
  - Derivatives of **KE** ⇒ how kinetic energy (mass or velocity) changes affect response.
  - Derivatives of **FD** ⇒ how coefficient of friction changes affect response.

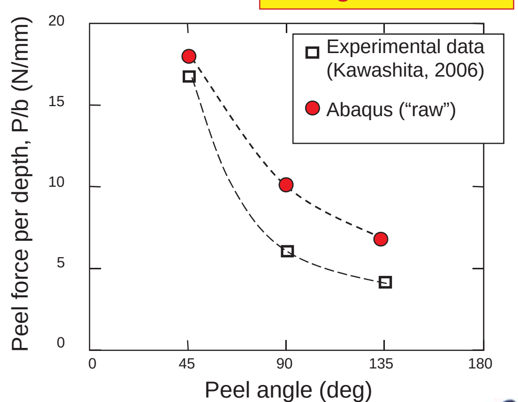
- These estimates of change or influence are most accurate if the structure's components behave as springs in parallel.
  - Assessing energy derivatives for structures with components that act like springs in series is also helpful, but more difficult and typically less accurate since obtaining complimentary energy is often impossible and must be estimated.

FE Modeling of Peeling Via Cohesive Elements



- Abaqus/Explicit model
  - Plasticity plus cohesive elems.
  - Adaptive mass scaling of cohesive elements
  - Solution/experiment correlation is “so so”

Hey, Peeling is a Tough Problem



FE Modeling of Peeling Via Cohesive Elements

Improving Quasi-Static Estimate

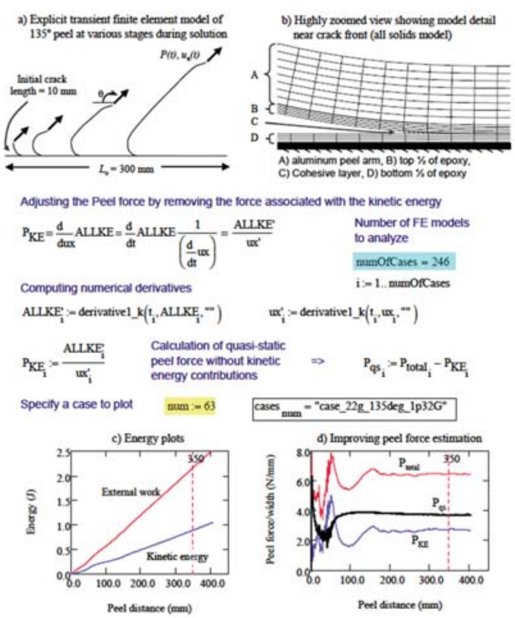
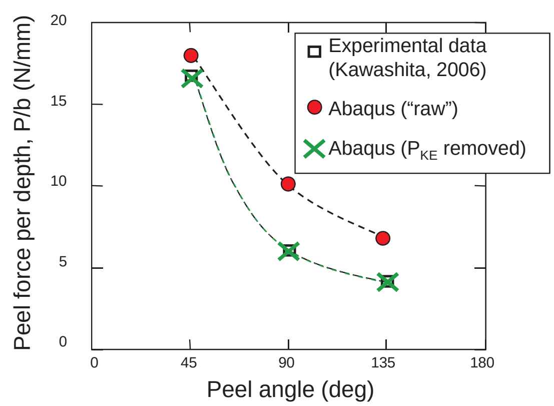


Figure 6: Significantly enhancing analysis accuracy and solution speed by adjusting the calculated peel load using derivatives of computed energy terms.

In this example, correcting for so-called “kinetic energy force” in quasi-static explicit FEA model makes significant improvement!



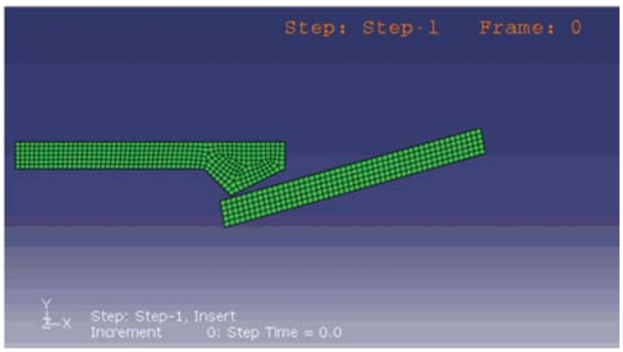
Ref:
Diehl, T., “Integrating Theory, Experiments, and FEA to Solve Challenging Nonlinear Mechanics Problems,” *Abaqus Users' Conference Proceedings*, 2008.

Quasi-Static Snap-Fit Via Explicit Dynamics FEA

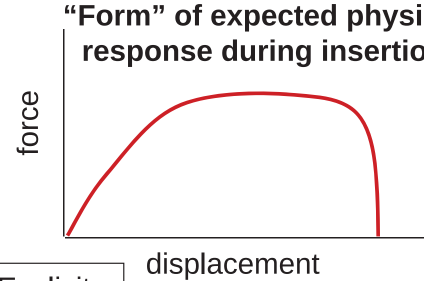
Preliminary scouting analysis of a Snap-Fit design

- Understand sensitivity to lower arm angle and influence of friction.
- Simulate both insertion and retraction
  - Implicit FEA will have difficulty with “snap” and retraction, so simulate with Explicit FEA.

3 angles: θ_o, θ_o+5º, θ_o-5º



“Form” of expected physical response during insertion



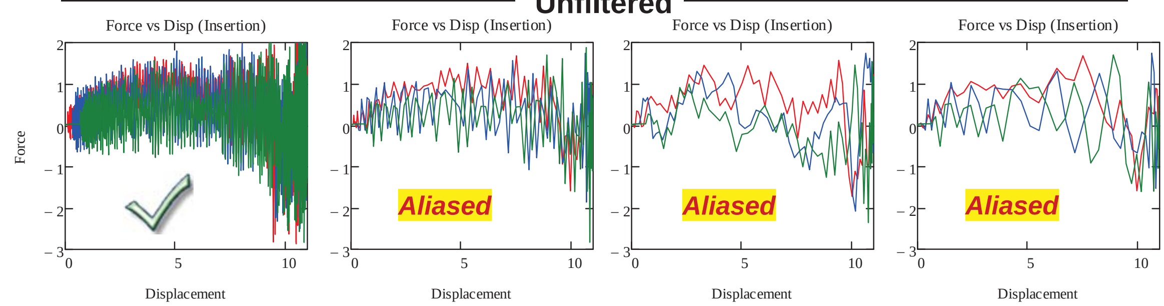
FEA via Abaqus/Explicit,
Filtering via Kornucopia®

Quasi-Static Snap-Fit Via Explicit Dynamics FEA

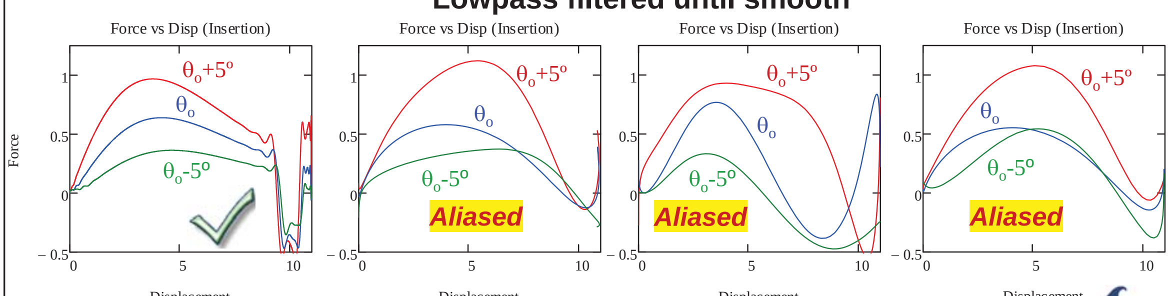
Insertion Results Using Different Output Requests

Every increment (~35,000 incs)	100 increments	80 increments	50 increments

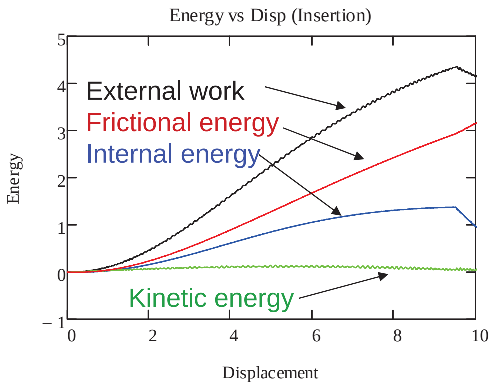
Unfiltered

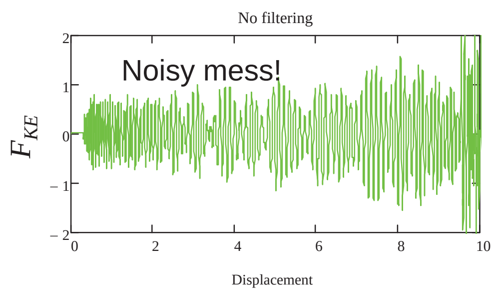


Lowpass filtered until smooth



Quasi-Static Snap-Fit Via Explicit Dynamics FEA
Energy Methods, Derivatives, and DSP to Improve Analysis





Below are two "forces" derived from derivatives of energy quantities. In certain cases, such quantities can be used to enhance results interpretations.

Kinetic Force – load associated with moving mass.

$$\boxed{F_{KE} = \frac{\partial(U_{KE})}{\partial(u)}}$$

U_{KE} – kinetic energy
u - displacement

Frictional Force – portion of total load caused by friction.

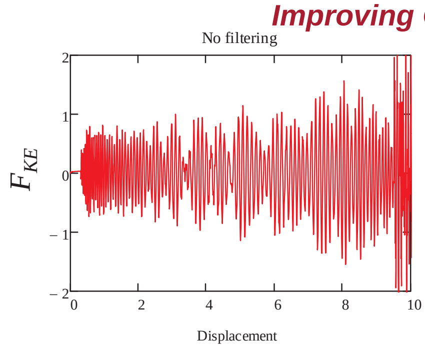
$$\boxed{F_{fric} = \frac{\partial(U_{friction})}{\partial(u)}}$$

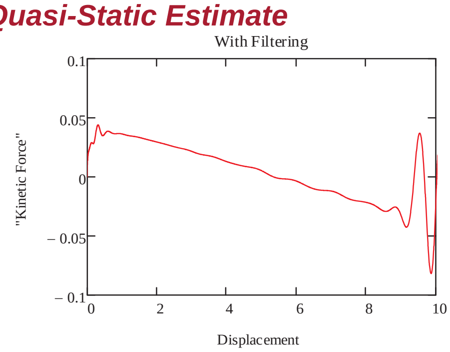
$U_{friction}$ – total energy consumed by friction

Quasi-Static Snap-Fit Via Explicit Dynamics FEA
Energy Methods, Derivatives, and DSP to Improve Analysis

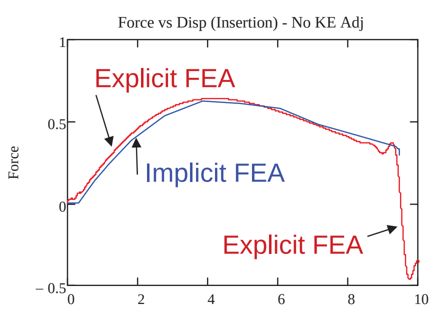
Improving Quasi-Static Estimate

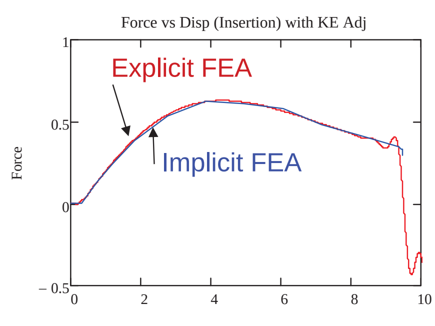
Case: θ_o





Filtered via bi-directional lowpass 6^{th} order butterworth, $f_c = 0.005 \cdot f_s$



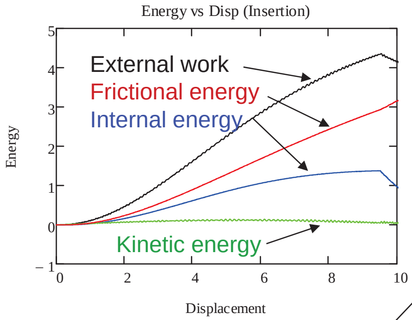


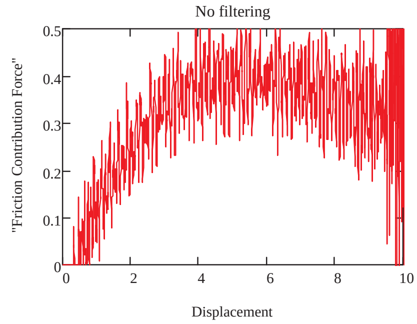
The small difference between the implicit FEA and explicit FEA results is shown to be a result of "moving mass".

Quasi-Static Snap-Fit Via Explicit Dynamics FEA

Energy Methods, Derivatives, and DSP to Improve Analysis

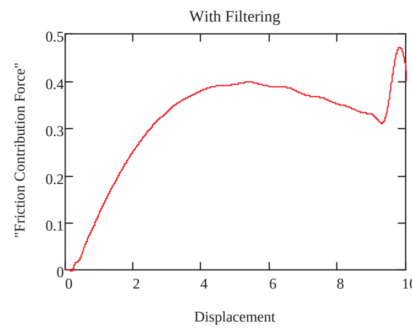
Estimating Frictional Influence





Case: θ_o

$$F_{fric} = \frac{\partial\left(U_{friction}\right)}{\partial(u)}$$



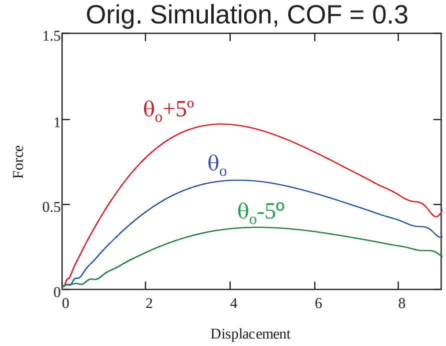
Filtered via bi-directional lowpass 6^{th} order butterworth, $f_c=0.005\cdot f_s$

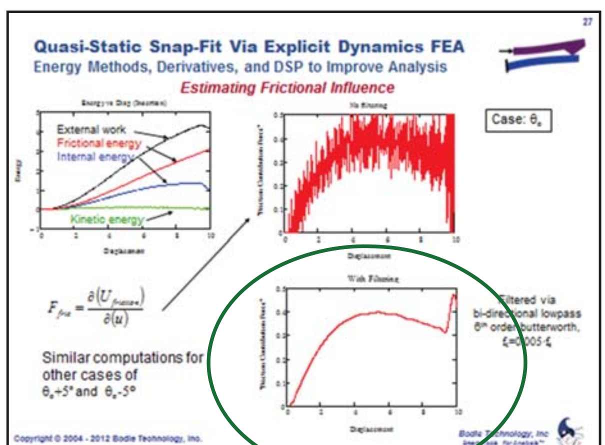
Similar computations for other cases of θ_o+5º and θ_o-5º

Quasi-Static Snap-Fit Via Explicit Dynamics FEA

Energy Methods, Derivatives, and DSP to Improve Analysis

Estimating Frictional Influence





Prediction equation

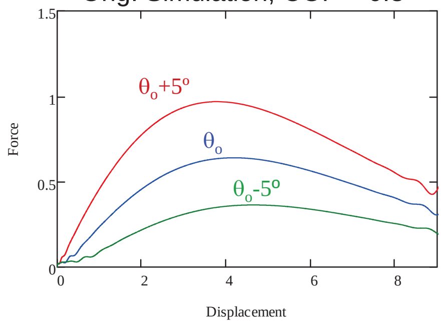
$$F_{total}\Big|_{COF=new} = F_{total}\Big|_{COF=0.3} + F_{fric}\Big|_{COF=0.3} \cdot \left(\frac{COF_{new}}{COF{=}0.3} - 1\right)$$

Quasi-Static Snap-Fit Via Explicit Dynamics FEA

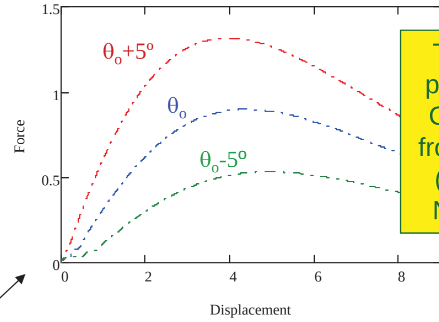
Energy Methods, Derivatives, and DSP to Improve Analysis

Estimating Frictional Influence

Orig. Simulation, COF = 0.3



Energy Derivative *Predictions*, COF = 0.5

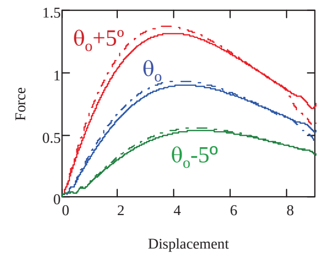


Three FAST predictions of COF change from 0.3 to 0.5 (FEA model NOT re-run)

Prediction equation

$$F_{total}\Big|_{COF=new} = F_{total}\Big|_{COF=0.3} + F_{fric}\Big|_{COF=0.3} \cdot \left(\frac{COF_{new}}{COF=0.3} - 1\right)$$

Validation for COF=0.5

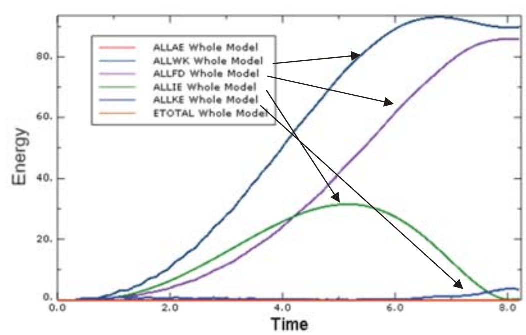


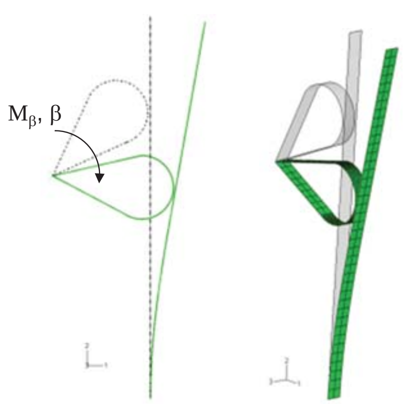
-.-.- prediction
—— actual FEA

Validating predictions by running actual full FEA models for all three angle cases with COF = 0.5

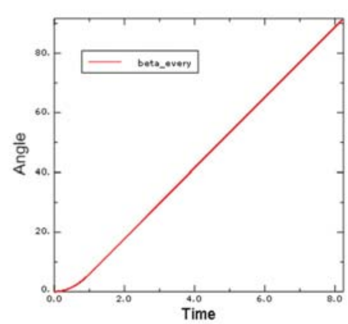
Getting More Out of Models

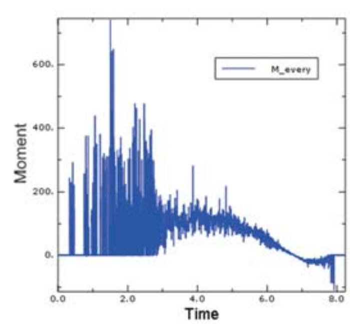
- Starting with this:
  - ***Running model only <u>ONCE</u>.***





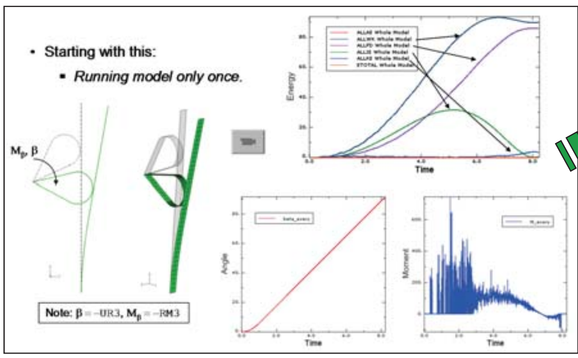
Note: $\beta = -$UR3, $M_\beta = -$RM3



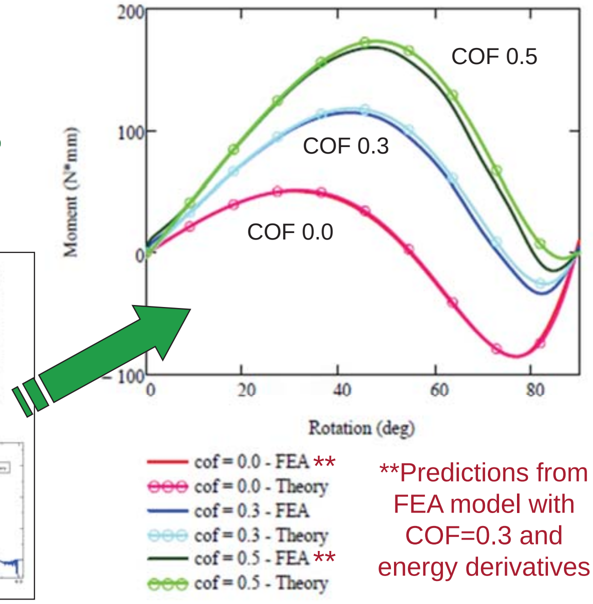


Getting More Out of Models

- We get these results by use of DSP and Energy Derivatives!
  - *Accurate results that match benchmark*
  - ***Prediction of friction influence WITHOUT additional runs of model!***



COF Predictions from ONE Run of Model



Conclusions

- Energy Balance MUST be satisfied
  - Confirming *ETOTAL = Constant* is a key sanity check for ALL models
- Assessing Kinetic Energy relative to Work and IE is important for Quasi-static Explicit Dynamics models
- Many engineers doing FEA and/or physical testing underutilize Energy Methods
- Energy Methods can be effectively used to estimate impact behavior with Quasi-static models or experiments.
  - Powerful technique, but limitations apply (read fine print!)
- Causality is derived from energy derivatives
  - Most accurate for structures with behavior similar to springs in parallel
  - Enables computing "Hit List" of key components/materials from single run
  - Enables fast prediction equations (good for linear and nonlinear problems)

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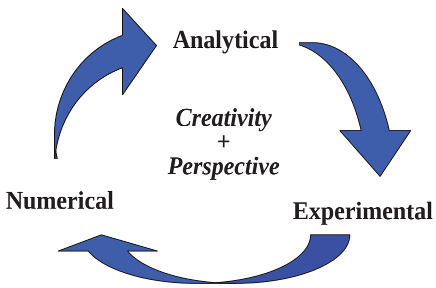

Want to learn more about these Advanced Energy Methods?
The topic is covered in greater detail (including workshops) in our short course

Analyzing Noisy Data via Filtering and DSP

Instructor's Profile –Ted Diehl, Ph.D.

- **Expert in Nonlinear Mechanics with 20+ Years Experience**
  - Experimental, computational, and theoretical approaches
  - Led nonlinear mechanics efforts at Kodak, Motorola, and DuPont
- **Engineering Tools**
  - Mathcad & Kornucopia®, Abaqus nonlinear FEA, and experimental methods
- **Engineering Success in Industry**
  - NASA spacecraft
  - Cell-phone impact
  - Peeling mechanics
  - Paper motion in copiers
  - Simulating fabrics
  - Flexible structures
  - Nonlinear nip mechanics
  - Ballistic protection
  - Nonlinear materials
- **Created unique DSP algorithms**
  - Enhance analysis of noisy data from experiments & Explicit Dynamics FEA
- **Developer of Kornucopia® and President of Bodie Technology Inc.**
  - ***Smart-Tools for Analyzing Noisy & Challenging Problems™***